Application and power of tests for homogeneity of variances

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Abstract – **The methods of statistical modeling are used to investigate the distribution of test statistics for homogeneity of means. The results of the comparative power analysis of the tests for homogeneity of means depending on the alternative hypotheses are given. The conclusions about preferential use of the test are provided as well. The possibility of tests application to the samples that do not fit the normal distribution and analysis of tests statistic distribution in such condition is implemented. The results obtained should contribute to the correct application of the tests.**

Key words – **tests, homogeneity test, power of test, statistics modeling.**

I. INTRODUCTION

CRITERIA for testing homogeneity hypotheses are used in many applications related to the analysis of experimental results. It may be about testing hypotheses about the homogeneity of the distribution laws that correspond to the samples being analyzed, or about the homogeneity of mathematical expectations (on the equality of means), or on the uniformity of variances.

All the existing tests for testing hypotheses about the homogeneity of laws are nonparametric. To test hypotheses about the homogeneity of mathematical expectations and the homogeneity of variances, parametric and nonparametric tests are used. An assumption that makes it possible to use parametric tests (the possibility of using classical results) is the observance of the observed random variables (measurement errors) to the normal distribution law.

It is known that the parametric tests for homogeneity of the averages are stable to the violation of the normality assumption. Classical results on the distributions of statistics of parametric tests for homogeneity of means can be used in violation of the standard assumption of normality provided that the form of the law to which the samples belong is not a law with very "heavy tails" or essentially asymmetric [1].

On the contrary, the parametric tests for the homogeneity of variances are extremely sensitive to the smallest deviations of the observed random variables from the normal law. If this assumption is violated, the conditional distributions of the statistics of the tests for the validity of the hypothesis being tested are, as a rule, greatly altered.

This remark also applies to such a "stable" criterion as the Levene criterion [2] and its modifications [3]. Since the random variables observed in various applications and the measurement errors are not always subject to normal law, the application of classical results under such conditions can lead to incorrect conclusions. In the case of non-parametric tests for checking the homogeneity of variances, it does not matter which law the analyzed samples are, but it is assumed that they belong to the same kind of law. The need for this assumption significantly limits the possibilities of nonparametric tests [4]. It should also be noted that at present, many non-parametric tests, with one exception, are limited to tests intended for the analysis of 2 samples. Studies have shown that non-parametric tests, as a rule, yield in parametric power.

The latter circumstance pushes us to realize the possibility of applying parametric tests for the homogeneity of variances under the conditions of violation of the standard assumption of normality, which is not a super complicated task when using statistical modeling methods [5] and related software [6]. The results of previous studies of the properties of parametric and nonparametric tests for the homogeneity of dispersions and their power [7, 8, 4, 9, 10, 11] entered the guide [12].

In this paper, the main attention is paid to the study of the properties and power of two parametric tests for the homogeneity of variances not included in the earlier prepared manual [12] (Miller tests [13] and Layard [14]), a also a comparative analysis of the power of the tests, including under conditions of violation of the standard assumption of normality.

II. RESEARCH MILLER AND LAYARD TESTS

In the tests for checking the homogeneity of variances, the hypothesis of the constancy of variances k samples has the form

$$H_0\colon \sigma_1^2 = \sigma_2^2 = \ldots = \sigma_k^2 , \qquad (1)$$

The hypothesis competing with it

$$H_1\colon \sigma_{i_1}^2 \neq \sigma_{i_2}^2 , \qquad (2)$$

where the inequality holds, at least for one pair of indices i_1, i_2. When researching statistical distributions and evaluating the power of criteria in conditions of limited sample volumes, analytical methods usually do not work, and therefore statistical methods of modeling rely on computer technology. In this case, also in the study of the distributions of statistics of the criteria considered and the evaluation of the power of the criteria for various competing hypotheses, the statistical modeling technique [5] and developed on the basis of [15] program system ISW (Interval

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statistics on Windows) [6]. At the same time, the volume of simulated samples of the statistics was studied a $N=10^6$ value. At such N a difference between the true law of distribution of statistics and the empirical model modulated does not exceed 10^3.

Investigations of the distributions of statistics of criteria for the validity of competing hypotheses were carried out under both normal and other laws. In particular, the power was also investigated in the case of the model samples being assigned to the generalized normal law with density

$$De(\theta_2)=f(x;\theta_0,\theta_1,\theta_2)=\frac{\theta_2}{2\theta_1\Gamma(1/\theta_2)}\exp\left(-\left(\frac{|x-\theta_0|}{\theta_1}\right)^{\theta_2}\right) \quad (3)$$

for different values of the form parameter θ_2. This family can be a good model for the laws of error distribution of various measuring systems. Distribution $\mathrm{De}(\theta_2)$ includes as special cases the Laplace distribution $(\theta_2=1)$ and normal $(\theta_2=2)$. Family (3) allows you to specify different symmetric distribution laws that differ to some extent from the normal: the smaller the value of the form parameter θ_2, the "heavier" the distribution tails $\mathrm{De}(\theta_2)$, The larger the parameter, the easier the tails.

A. *The Miller test*

Miller [13] proposed a criterion for homogeneity of variances, based on F –Fisher transformation for selective variances. Layard [14] summarized Miller's two-sample criterion for the case k samples. Statistics k- sampling Miller test has the form

$$M=\frac{\sum_{i=1}^{k} n_i\left(\bar{U}_{i\bullet}-\bar{U}_{\bullet\bullet}\right)^2 \Big/ (k-1)}{\sum_{i=1}^{k}\sum_{j=1}^{n_i}\left(U_{ij}-\bar{U}_{i\bullet}\right)^2 \Big/ (n-k)}, \quad (4)$$

where

$$U_{ij}=n_i\ln S_i^2-(n_i-1)\ln S_{i(j)}^2;$$

$$S_i^2=\frac{1}{n_i-1}\sum_{j=1}^{n_i}\left(x_{ij}-\bar{x}_i\right)^2;\ \bar{x}_i=\frac{1}{n_i}\sum_{j=1}^{n_i}x_{ij};$$

$$S_{i(j)}^2=\frac{1}{n_i-2}\sum_{l\neq j}\left(x_{il}-\bar{x}_{i(j)}\right)^2;\ \bar{x}_{i(j)}=\frac{1}{n_i-1}\sum_{l\neq j}x_{il};$$

$$\bar{U}_{i\bullet}=\frac{1}{n_i}\sum_{j=1}^{n_i}U_{ij};\ \bar{U}_{\bullet\bullet}=\frac{1}{n}\sum_{i=1}^{k}\sum_{j=1}^{n_i}U_{ij};\ n=\sum_{i=1}^{k}n_i.$$

If the hypothesis to be tested is valid H_0 on the homogeneity of variances and the fulfillment of assumptions about the attribution of samples to normal laws, statistics (4) must obey F –distribution with the numbers of degrees of freedom $(k-1)$ and $(n-k)$. The hypothesis to be tested is rejected for large values of statistics.

Note that for small sample sizes, the distribution $G(M_n|H_0)$ of statistics (4) Miller significantly differs from the corresponding F – distribution. In Fig.1 the deviation of the real distribution is shown $G(M_n|H_0)$ from the F –distribution for $k=4$ at volumes of compared samples $n_i=10$. Really the deviation of the distribution $G(M_n|H_0)$ statistics from the Fisher distribution with $(k-1)$ and $(n-k)$ degrees of freedom can be neglected when $n_i>40\div50$.

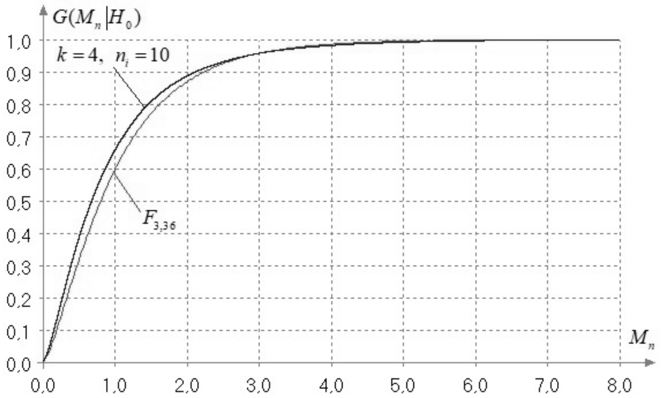

Fig.1. The deviation of the distribution of the statistics (4) from the distribution $F_{k-1,\,n-k}$

The distribution of the statistics (4) are sensitive to the violation of the assumption of the normality of the analyzed samples.

A. *The Layard test*

Layard [14] presented a criterion with statistics in which the kurtosis function of several samples is used to check the uniformity of variances.

The statistics of Layard's criterion is given by

$$L=\sum_{i=1}^{k}(n_i-1)\left(\ln S_i^2-T\right)^2\Big/\delta^2, \quad (5)$$

where $\delta^2=2+\gamma\left[1-k/n\right]$;

$$T=\left[\sum_{i=1}^{k}(n_i-1)\ln S_i^2\right]\Big/(n-k);\ n=\sum_{i=1}^{k}n_i;$$

$$\gamma=\frac{1}{n}\sum_{i=1}^{k}n_i^2\left[\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^4\Big/\left(\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^2\right)^2\right]-3. \quad (6)$$

Here γ –weighted average of kurtosis coefficients k samples.

The author of the criterion chose to use in statistics (5) a slightly different estimate of the kurtosis coefficient:

$$\hat{\gamma}=n\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^4\Big/\left[\sum_{i=1}^{k}\sum_{j=1}^{n_i}(x_{ij}-\bar{x}_i)^2\right]^2-3. \quad (7)$$

The hypothesis to be tested is rejected for large statistics (5), which, under the validity of the hypothesis being tested, about the homogeneity of variances and the assumption that samples belong to normal laws asymptotically obeys from the distribution χ^2_{k-1}. However, it should be noted that the convergence of the distribution of statistics $G(L_n|H_0)$ the distribution χ^2_{k-1} slow enough. Really rejection $G(L_n|H_0)$ at distribution χ^2_{k-1} for example, with $k=2$ can be neglected only when $n_i > 300$ (In Fig. 2). Note also that to calculate the statistics (5) it is preferable to use the estimate (7), since in the case of applying the stimate (6) convergence to $G(L_n|H_0)$ distribution χ^2_{k-1} is worse.

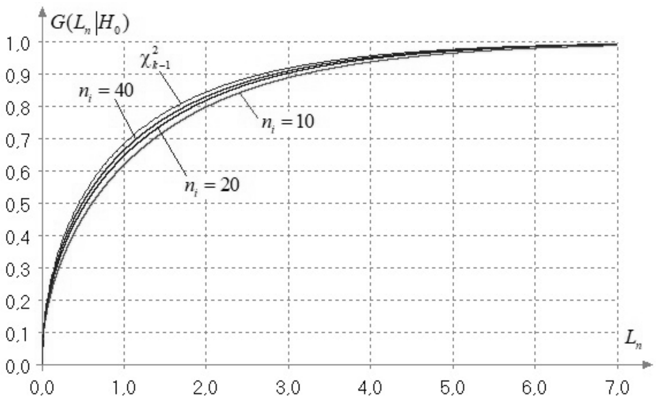

Fig.1. The convergence of the distribution of the statistics (5) to the χ^2_{k-1} distribution.

In terms of its asymptotic properties and power, the Layard criterion is very close to the Miller criterion and slightly exceeds the latter in power only for small sample sizes.

III. Comparative Analysis of the Powers of Tests

In the process of investigating the distributions of statistics (4) and (5) and estimating the power of the criteria for the homogeneity of variances, some features that could influence the formation of conclusions using the appropriate criteria were recorded.

This paper complements the studies concentrated in [12], where a comparative power analysis was performed and the properties of a number of parametric and nonparametric criteria were investigated, including under conditions of violation of standard assumptions.

In this case, by statistical modeling methods, the power of the considered criteria of homogeneity of means for the case of 2 samples was investigated with respect to three competing hypotheses:

$H_1: \sigma_2 = 1.1\sigma_1$; $H_2: \sigma_2 = 1.2\sigma_1$; $H_3: \sigma_2 = 1.5\sigma_1$.

Taking into account previous works, the results of the studies are presented in [12], in Tables 1-3 all the criteria considered (parametric: Bartlett [16], Cochran [17], Hartley [18], Fisher, Levene [2], Neumann–Pearson [19], O'Brien [20], Link (Range relations) [21], Newman (the standardized range) [22], Bliss-Cochran-Tukey [23], Cadwell-Leslie-Brown [24], Miller [13, 14], Layard [14], Z-test of Overall–Woodward [25] and the modified Z-test [26]) and non-parametric – Ansari–Bradley [27], Mood [28], Siegel–Tukey [29], Klotz [30], Fligner–Killeen [31]) ranked in descending order relative to competing hypotheses H_1, H_2, H_3 with probability of errors of the first kind $\alpha = 0.1$.

In the tables below, with power ratings for the names of the criteria, the following notations were used: B - Bartlett, C - Cochran, H - Hartley, F - Fisher, NP – Neumann–Pearson, Z – Z-test of Overall–Woodward, ZM - modified Z-test, Ld – Layard, Mr – Miller, OB – O'Brien, Kl – Klotz, Le – Levene, FK – Fligner–Killeen, M-Mood, ST – Siegel–Tukey, AB – Ansari-Bradley, New – Newman, BCT – Bliss–Cochran-Tukey, CLB - Cadwell-Leslie-Brown, Lk - Linka.

TABLE I
Estimates of power of tests for $H_1: \sigma_2 = 1.1\sigma_1$

Test	n_i				
	10	20	40	60	100
B, C, H, F, NP, Z	0.112	0.128	0.157	0.188	0.246
Ld	0.110	0.125	0.155	0.185	0.243
Mr	0.110	0.125	0.154	0.185	0.243
ZM, OB	0.109	0.125	0.154	0.184	0.243
Kl	0.109	0.123	0.151	0.181	0.236
Le	0.110	0.123	0.150	0.176	0.228
FK	0.108	0.121	0.147	0.172	0.223
M	0.108	0.120	0.143	0.166	0.212
New	0.111	0.123	0.143	0.159	0.186
AB	0.109	0.125	0.138	0.154	0.190
ST	0.107	0.120	0.137	0.154	0.190
BCT, CLB, Lk	0.111	0.119	0.133	0.141	0.154

TABLE II
Estimates of power of tests for $H_2: \sigma_2 = 1.2\sigma_1$

Test	n_i				
	10	20	40	60	100
B, C, H, F, NP, Z	0.144	0.199	0.304	0.401	0.564
Ld	0.139	0.193	0.295	0.391	0.557
Mr	0.137	0.191	0.294	0.391	0.557
ZM, OB	0.134	0.188	0.292	0.389	0.555
Kl	0.133	0.183	0.280	0.376	0.540
Le	0.135	0.184	0.276	0.363	0.515
FK	0.131	0.177	0.266	0.351	0.503
M	0.130	0.172	0.253	0.331	0.470
New	0.140	0.183	0.251	0.304	0.386
AB	0.133	0.171	0.232	0.290	0.405
ST	0.128	0.166	0.228	0.290	0.405
BCT, CLB, Lk	0.139	0.171	0.216	0.246	0.289

TABLE III
ESTIMATES OF POWER OF TESTS FOR $H_3: \sigma_2 = 1.5\sigma_1$

Test	n_i				
	10	20	40	60	100
B, C, H, F, NP, Z	0.312	0.532	0.806	0.926	0.991
Ld	0.289	0.503	0.787	0.918	0.990
Mr	0.281	0.500	0.786	0.918	0.990
OB	0.266	0.490	0.783	0.917	0.990
ZM	0.265	0.489	0.781	0.916	0.990
Kl	0.258	0.463	0.754	0.900	0.987
Le	0.269	0.471	0.746	0.888	0.981
FK	0.249	0.442	0.719	0.870	0.977
M	0.243	0.424	0.688	0.842	0.964
New	0.296	0.473	0.682	0.796	0.901
AB	0.242	0.392	0.616	0.768	0.926
ST	0.231	0.384	0.613	0.768	0.926
BCT, CLB, Lk	0.285	0.425	0.584	0.674	0.776

When analyzing the power of the criteria in the case where the number of samples is greater than two, as competing hypotheses, situations were considered where $k-1$ sample belonged to the law with some $\sigma = \sigma_1$, and one of the samples, for example, with the number k belonged to a law with a different meaning σ ($H_1: \sigma_k = 1.1\sigma_1$; $H_2: \sigma_k = 1.2\sigma_1$; $H_3: \sigma_k = 1.5\sigma_1$).

The results of a comparative analysis of the power of criteria relative to competing hypotheses H_1, H_2, H_3 here $k = 3$ and $k = 5$ are presented in tables 4-6. It can be seen that in the case of a number of samples, more than two of the Layard test lose their advantage over the Miller test.

TABLE IV
ESTIMATES OF POWER OF TESTS FOR
$H_1: \sigma_2 = 1.1\sigma_1$, $n_i = 100$, $i = \overline{1,k}$

Test	α					
	0.1	0.05	0.01	0.1	0.05	0.01
	$k = 3$			$k = 5$		
C	0.250	0.161	0.056	0.241	0.156	0.056
OB	0.243	0.153	0.051	0.230	0.144	0.048
Z	0.243	0.153	0.051	0.227	0.141	0.046
NP, B	0.242	0.152	0.049	0.224	0.138	0.044
ZM	0.240	0.150	0.048	0.223	0.137	0.044
H	0.239	0.148	0.046	0.219	0.133	0.040
Mr	0.237	0.146	0.045	0.216	0.129	0.038
Ld	0.236	0.146	0.044	0.215	0.128	0.037
Le	0.225	0.139	0.043	0.209	0.127	0.039
FK	0.222	0.137	0.042	0.206	0.124	0.038
CLB	0.149	0.083	0.021	0.139	0.075	0.018
BCT	0.147	0.082	0.021	0.136	0.075	0.019

TABLE V
ESTIMATES OF POWER OF TESTS FOR
$H_2: \sigma_2 = 1.2\sigma_1$, $n_i = 100$, $i = \overline{1,k}$

Test	α					
	0.1	0.05	0.01	0.1	0.05	0.01
	$k = 3$			$k = 5$		
C	0.997	0.994	0.974	0.998	0.997	0.987
OB	0.996	0.990	0.961	0.997	0.994	0.976
Z	0.996	0.991	0.964	0.997	0.993	0.974
NP, B	0.996	0.990	0.962	0.996	0.992	0.970
ZM	0.995	0.989	0.955	0.996	0.991	0.967
H	0.995	0.988	0.947	0.995	0.989	0.955
Mr	0.995	0.987	0.946	0.994	0.987	0.949
Ld	0.994	0.987	0.941	0.994	0.986	0.942
Le	0.990	0.979	0.926	0.991	0.982	0.944
FK	0.987	0.973	0.909	0.988	0.977	0.928
BCT	0.820	0.728	0.501	0.829	0.742	0.524
CLB	0.795	0.691	0.444	0.783	0.675	0.432

TABLE VI
ESTIMATES OF POWER OF TESTS FOR
$H_3: \sigma_2 = 1.5\sigma_1$, $n_i = 100$, $i = \overline{1,k}$

Test	α					
	0.1	0.05	0.01	0.1	0.05	0.01
	$k = 3$			$k = 5$		
C	0.997	0.994	0.974	0.998	0.997	0.987
OB	0.996	0.990	0.961	0.997	0.994	0.976
Z	0.996	0.991	0.964	0.997	0.993	0.974
NP, B	0.996	0.990	0.962	0.996	0.992	0.970
ZM	0.995	0.989	0.955	0.996	0.991	0.967
H	0.995	0.988	0.947	0.995	0.989	0.955
Mr	0.995	0.987	0.946	0.994	0.987	0.949
Ld	0.994	0.987	0.941	0.994	0.986	0.942
Le	0.990	0.979	0.926	0.991	0.982	0.944
FK	0.987	0.973	0.909	0.988	0.977	0.928
BCT	0.820	0.728	0.501	0.829	0.742	0.524
CLB	0.795	0.691	0.444	0.783	0.675	0.432

IV. APPLICATION OF TESTS FOR VIOLATION OF STANDARD ASSUMPTIONS

The violation of the assumption of normality leads to significant changes in the distributions $G(S|H_0)$ statistics of parametric criteria. To a lesser extent, this concerns O'Brien's criterion, the modified Overall-Woodward Z-criterion, and Levene's criterion modification. However, the distributions of statistics and these three criteria deviate so much from those that occur under standard assumptions, which can not be neglected.

The correctness of the conclusions on the criteria used depends on how accurately knowledge of the distribution

$G(S|H_0)$ correspond to the real conditions characterizing the analyzed measurement results. If we know the form of the distribution law to which the analyzed samples belong, or we can determine it based on the available experimental data, then in order to form a correct derivation we need to find only the distribution $G(S|H_0)$ in these real conditions.

At present, due to the sharp increase in the capabilities of computer technology in software systems of statistical analysis, the role of the use of computer methods for studying regularities increases substantially. When the distribution of the statistics of the criterion used to test the hypothesis is unknown at the time the test begins, it becomes possible to study the distribution of statistics in real-time hypothesis testing. In particular, in the interactive mode, it is possible to investigate the unknown distribution of the statistics of any criterion for the homogeneity of variances, depending on the sample size, for those values n_i ,which correspond to the analyzed samples, and to estimate the level of significance obtained by modeling the empirical distribution of statistics p_{value}. With this approach, the empirical distribution necessary for testing the hypothesis $G_N(S_n|H_0)$ statistics of the corresponding criterion is constructed as a result of statistical modeling with an accuracy that depends on the number of experiments N in the Monte Carlo method. The implementation of such an interactive mode requires the availability of advanced software that allows [6] in order to accelerate the parallelization of modeling processes and to attract available computing resources. In parallelizing conditions, the time for constructing the distribution $G_N(S_n|H_0)$ statistics of the criterion is not very noticeable against the background of a complete solution of the problem of statistical analysis.The use of the interactive mode for the study of statistical distributions opens the possibility of applying the criteria in conditions of violation of the standard assumption that the results of measurements are normal [32].

V. POWER OF TESTS WITH VIOLATION OF THE NORMALITY PREREQUISITES

Under the conditions of violation of the standard assumption of normality, the power of the criteria was investigated in the situation when samples belonged to the generalized normal law with density (3) for various values of the form parameter θ_2. Further in the text and tables the designation $\mathrm{De}(\theta_2)$ corresponds to a distribution of the form (3) with the corresponding value of the form parameter θ_2.

Tables 7-9 give estimates of the power of criteria relative to competing hypotheses H_2 and H_3, obtained in the case of sample membership to the generalized normal law (3) with different values of the shape parameter for sample volumes $n_i = 100$, $i = \overline{1..k}$. In the tables, the values α are given in%, the power estimates in the form $(1-\beta)*1000$. In the tables, the criteria are ordered in descending order of power, which they have in the case of a normal law (see at $\mathrm{De}(\theta_2)$). As can be seen, the order of preference for the criteria varies depending on the severity of the tails. The Miller and Layard tests considered in this paper are close in power, they are inferior only to a group of criteria equivalent under the conditions of the standard assumption of normality.

It may be noted that only the non-parametric Fligner–Killeen test allows one to analyze the number of samples $k > 2$. In this case, if the samples belong to the laws with "heavy tails", this criterion has an advantage in power over the others. However, it should be borne in mind that the distribution of the statistics of the criterion still depends on which laws belong to the samples [12], that is, the "nonparametric" criterion is somewhat relative. This means that the correctness of the conclusions can be ensured only by using, during the hypothesis testing, interactive modeling of the distribution of statistics in the specific conditions of the application, as is done in [6].

As further plans for the situation of the number of samples $k > 2$ one can consider the possibility of applying to each pair of samples the two-sample tests of Mood, Siegel–Tukey and Ansari-Bradley, and then making a decision on the minimum value of the achieved significance level for all pairs.

VI. CONCLUSION

The methods of statistical modeling are used to investigate the distribution of test statistics for homogeneity of variances. The results of the comparative power analysis of the tests for homogeneity of variances depending on the alternative hypotheses are given. Within the developed software system ISW [6] an interactive mode of research of distribution of statistics is implemented, allowing to estimate the reached level of significance p_{value} in the situation of violation of standard assumptions or in the case of an unknown distribution of statistics, given only by a table of percentage points. This makes the statistical conclusion about the results of the hypothesis test more informative and more justified.

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TABLE VII

THE POWER OF THE TEST FOR THE COMPETING HYPOTHESIS H_2 IN THE CASE OF THE ACCESSORIES OF SELECTIONS TO THE FAMILY DISTRIBUTION (3) WITH VARIOUS VALUES OF THE FORM'S PARAMETER θ_2 AT $k=2$, $n_i=100$, $i=\overline{1,k}$

Test	*De*(0.5)			*De*(0.5)			*De*(2)			*De*(3)			*De*(4)			*De*(5)		
	α																	
	10	05	01	10	05	01	10	05	01	10	05	01	10	05	01	10	05	01
B, C, H, F, NP, Z	162	091	022	317	213	078	**564**	**438**	**218**	**689**	**570**	**326**	754	644	**398**	791	689	446
Mr	167	097	028	309	207	075	557	429	207	687	566	321	755	645	397	793	691	450
Ld	179	106	031	322	218	079	557	428	205	681	557	307	745	630	375	783	674	423
ZM	167	096	024	310	206	073	555	427	205	685	564	319	752	642	395	790	688	446
OB	176	103	029	322	216	078	555	427	205	680	556	306	745	630	374	782	674	420
Kl	224	139	044	346	237	090	540	412	196	673	549	302	**760**	**649**	394	**820**	**720**	**472**
Le	215	132	040	**356**	**245**	**093**	515	388	180	588	460	232	627	500	264	649	524	283
FK	**232**	**145**	**045**	344	234	088	503	376	173	604	477	244	669	544	300	713	593	344
M	222	138	043	324	218	081	468	344	152	558	431	214	618	492	261	659	536	298
ST, AB	213	131	041	296	196	070	405	287	119	470	348	157	513	388	184	542	416	204
New	144	080	020	224	141	047	386	276	116	527	405	200	638	517	288	720	608	370
BCT, CLB, Lk	128	069	016	173	101	028	289	190	068	417	299	127	540	415	203	650	527	292

TABLE VIII

THE POWER OF THE TEST FOR THE COMPETING HYPOTHESIS H_3 IN THE CASE OF THE ACCESSORIES OF SELECTIONS TO THE FAMILY DISTRIBUTION (3) WITH VARIOUS VALUES OF THE FORM'S PARAMETER θ_2 AT $k=2$, $n_i=100$, $i=\overline{1,k}$

Test	*De*(0.5)			*De*(0.5)			*De*(2)			*De*(3)			*De*(4)			*De*(5)		
	α																	
	10	05	01	10	05	01	10	05	01	10	05	01	10	05	01	10	05	01
B, C, H, F, NP, Z	388	266	095	827	734	501	**991**	**980**	**924**	**999**	**997**	**985**	**1.00**	**999**	**995**	**1.00**	**1.00**	**998**
Mr	392	282	120	800	700	459	990	977	910	999	997	983	1.00	999	994	1.00	1.00	997
Ld	445	328	146	830	737	498	990	977	906	999	996	978	1.00	999	992	1.00	1.00	996
ZM	400	283	107	804	699	447	990	976	906	999	997	982	1.00	999	994	1.00	1.00	997
OB	430	310	129	828	730	482	990	976	903	999	996	977	1.00	999	992	1.00	1.00	996
Kl	611	486	254	869	788	565	987	971	892	998	995	974	1.00	999	992	1.00	1.00	997
Le	579	452	226	**882**	**805**	**585**	981	960	866	993	984	934	996	991	957	998	993	967
FK	**634**	**508**	**272**	864	782	557	977	952	847	994	985	936	998	993	966	999	996	979
M	606	480	252	837	746	516	964	931	802	988	974	908	995	987	948	997	992	965
ST, AB	574	448	228	787	684	444	926	869	693	963	929	802	976	953	854	983	964	882
New	299	198	070	589	473	262	901	840	667	981	963	887	997	992	967	999	998	990
BCT, CLB, Lk	233	145	045	432	312	132	776	671	430	938	890	729	987	973	904	998	995	974

TABLE IX

THE POWER OF THE TEST FOR THE COMPETING HYPOTHESIS H_2 IN THE CASE OF THE ACCESSORIES OF SELECTIONS TO THE FAMILY DISTRIBUTION (3) WITH VARIOUS VALUES OF THE FORM'S PARAMETER θ_2 AT $k=5$, $n_i=100$, $i=\overline{1,k}$

Test	*De*(0.5)			*De*(0.5)			*De*(2)			*De*(3)			*De*(4)			*De*(5)		
	α																	
	10	05	01	10	05	01	10	05	01	10	05	01	10	05	01	10	05	01
C	134	070	015	306	208	081	**624**	**515**	**316**	**767**	**680**	**480**	**834**	**762**	**581**	**869**	**807**	**643**
OB	160	092	026	314	214	086	575	460	258	709	605	391	778	684	478	815	731	533
Z	141	074	016	297	197	073	565	445	241	702	592	371	772	673	457	811	722	512
ZM	148	081	019	289	190	070	554	433	228	697	587	364	770	672	454	810	721	514
B, NP	142	075	016	293	192	070	557	434	227	695	581	355	766	664	439	806	713	495
H	140	074	016	281	180	061	545	418	204	685	565	324	758	649	405	799	699	459
Mr	146	081	021	272	174	058	530	400	189	676	554	314	752	642	401	795	695	461
Ld	155	086	021	283	180	058	527	395	181	665	539	293	739	623	372	781	674	426
Le	197	119	036	**340**	**234**	**095**	513	390	197	591	471	260	633	516	298	657	542	323
FK	**212**	**128**	**039**	322	217	082	498	378	187	617	499	283	693	582	363	742	641	423
BCT	114	059	013	148	083	021	262	170	061	413	301	136	569	454	248	704	601	384
CLB	119	062	013	153	085	021	253	158	052	380	263	105	514	386	183	638	513	280

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