Models of statistical distributions of nonparametric goodness-of-fit tests in testing composite hypotheses of the generalized Weibull distribution

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ABSTRACT: In this paper there are presented results (tables of percentage points and statistic distribution models) for the nonparametric goodness-of-fit tests in testing composite hypotheses using the maximum likelihood method for parameters estimation for Generalized Weibull Distribution law. Statistic distributions of the nonparametric goodness-of-fit tests are investigated by the methods of statistical simulation.

1 THE FAMILY OF GENERALIZED WEIBULL DISTRBUTION IN RELIABILITY

Let as consider the sample $\mathrm{X} = (X_1, X_2, ..., X_n)^T$, we say that X_i follow the generalized Weibull distribution (GWD). The density function of the law is defined by:

$$f(x) = \frac{\theta_0}{\theta_1}\theta_2^{\theta_0} x^{\theta_0 - 1}\left(1+\left(\frac{x}{\theta_2}\right)^{\theta_0}\right)^{\frac{1}{\theta_1}-1} e^{1-\left(1+\left(\frac{x}{\theta_2}\right)^{\theta_0}\right)^{\frac{1}{\theta_1}}}, \quad (1)$$

where $x \geq 0$, θ_0, θ_1, $\theta_2 > 0$ (look fig. 1). The family (1) defines a set of different laws. Special cases of Generalized Weibull Distribution are: $\theta_1 = 1$ – the family of Weibull distribution; $\theta_0 = 1$, $\theta_1 = 1$ – the family of exponential distribution.

The distribution function is

$$F(x;\theta_0,\theta_1,\theta_2) = 1 - e^{1-\left(1+\left(\frac{x}{\theta_2}\right)^{\theta_0}\right)^{\frac{1}{\theta_1}}}.$$

The hazard function of GWD (look fig. 2) can be monotone increase ($\theta_0 > 1$, $\theta_0 > \theta_1$ and $\theta_0 = 1$, $\theta_1 < 1$), monotone decrease ($0 < \theta_0 < 1$, $\theta_0 < \theta_1$ и $0 < \theta_0 < 1$, $\theta_0 = \theta_1$), $\bigcap$ – shaped ($\theta_1 > \theta_0 > 1$), $\bigcup$ – shaped ($0 < \theta_1 < \theta_0 < 1$) and we have:

$$\lambda(x) = \frac{\theta_0}{\theta_1}\theta_2^{\theta_1} x^{\theta_0 - 1}\left(1+\left(\frac{x}{\theta_2}\right)^{\theta_0}\right)^{\frac{1}{\theta_1}-1}. \quad (2)$$

The GWD used in reliability and survival tasks along with lognormal distribution and inverse Gaussian distribution. As usual, in construction the models of laws for real observed random variables it is difficult to discriminate one laws from another and choose one of them. To certain degree these difficulties related with restricted facilities of application nonparametric goodness-of-fit tests with unknown statistic distribution for verification of composite hypotheses concerning GWD.

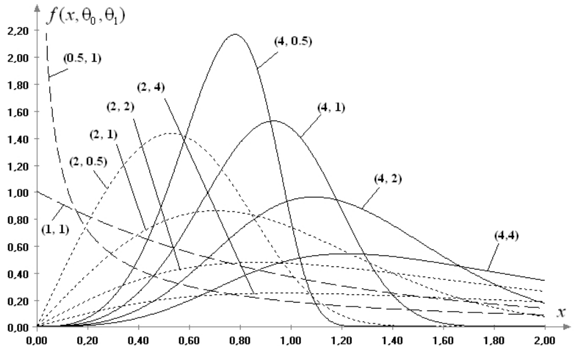

Fig.1. Dependence the density function (1) on parameters values (θ_0, θ_1).

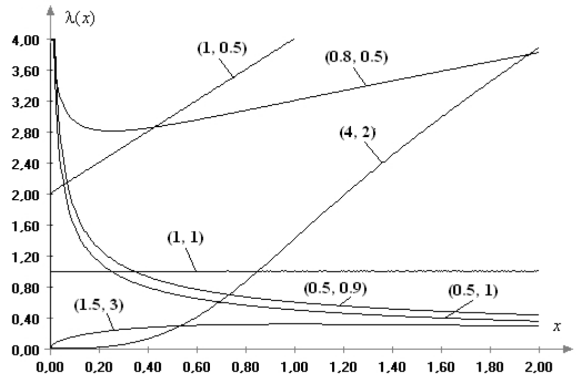

Fig.2. Dependence the hazard function (2) on parameters values (θ_0, θ_1).

2 NONPARAMETRIC GOODNESS-OF-FIT TESTS IN VERIFICATION SINGLE AND COMPOSITE HYPOTHESISES

One of the most popular statistic analysis problems in handling the results of experimental data is verification the agreement experimental distribution and theoretical one. There exist the verification of single hypothesis and composite hypothesis. Single hypothesis has the form $H_0: F(x) = F(x,\theta)$, $F(x,\theta)$ is probability distribution function, θ is known parameter value (θ is scalar parameter or vector parameter). In the case of single hypothesis marginal statistic distribution of nonparametric Kolmogorov, ω^2 Cramer-Von Mises-Smirnov, Ω^2 Anderson-Darling goodness-of-fit tests do not depend on view of observed distribution law and parameters values. These goodness-of-fit tests are "free from the distribution"

Composite hypotheses has the form H_0: $F(x) \in \{F(x,\theta), \theta \in \Theta\}$. In this case the estimate of distribution parameter $\hat{\theta}$ is calculated by the same sample, the nonparametric Kolmogorov, ω^2 Cramer-Von Mises-Smirnov, Ω^2 Anderson-Darling goodness-of-fit tests lose the property called "free from the distribution".

In Kolmogorov goodness-of-fit test the value

$$D_n = \sup_{|x|<\infty} \left| F_n(x) - F(x,\theta) \right|,$$

where $F_n(x)$ is the empirical distribution function, n is the sample size, is used in Kolmogorov test characterized a distance between the empirical and theoretical laws. In testing of hypotheses used a statistic with Bolshev correction (Bolshev, 1987) in the form (Bolshev and Smirnov, 1983)

$$S_K = \frac{6nD_n + 1}{6\sqrt{n}}, \qquad (3)$$

where $D_n = \max(D_n^+, D_n^-)$,

$$D_n^+ = \max_{1\le i\le n} \left\{ \frac{i}{n} - F(x_i,\theta) \right\},\ D_n^- = \max_{1\le i\le n} \left\{ F(x_i,\theta) - \frac{i-1}{n} \right\},$$

n is the sample size, $x_1, x_2, \ldots, x_n$ are sample values in increasing order is usually used. The distribution of statistic (3) obeys the Kolmogorov distribution law $K(S)$ (Bolshev and Smirnov, 1983) in testing simple hypotheses.

For verification of ω^2 Cramer-Von Mises-Smirnov goodness-of-fit test is used a statistic of the form

$$S_\omega = n\omega_n^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left\{ F(x_i,\theta) - \frac{2i-1}{2n} \right\}^2, \qquad (4)$$

and in test of Ω^2 Anderson-Darling type (Anderson and Darling, 1952, 1954) the statistic of the form

$$S_\Omega = -n - 2\sum_{i=1}^{n} \left\{ \frac{2i-1}{2n} \ln F(x_i,\theta) + \left(1 - \frac{2i-1}{2n}\right) \ln(1 - F(x_i,\theta)) \right\}. \qquad (5)$$

In verification a simple hypothesis statistic (4) obeys the $a1(S)$ distribution and statistic (5) obeys the $a2(S)$ distribution (see Bolshev and Smirnov, 1983).

In verification of composite hypotheses the conditional distribution law of the statistic $G(S|H_0)$ is affected by a number of factors: the form of the observed law $F(x,\theta)$ corresponding to the true hypothesis H_0; the type of the parameter estimation and the number of estimated parameters; sometimes it is a specific value of the parameter (e.g., in the case of gamma-distribution and beta-distribution families); the method of parameter estimation. The distinctions in the marginal distributions of the same statistics in testing simple and composite hypotheses are so significant that we cannot neglect them.

The paper (Kac *et al.*, 1955) was one of the first in investigating statistic distributions of the nonparametric goodness-of-fit tests with composite hypotheses. Then, for the solution to this problem, various approaches where used (Darling, 1955, 1957]), (Durbin, 1973, 1975, 1976), (Gihman, 1953, 1961), (Martinov,1978), (Pearson and Hartley, 1972), (Stephens, 1970, 1974), (Chandra *et al.*, 1981), (Tyurin, 1984), (Tyurin *et al.*, 1984), (Dzhaparidze and Nikulin, 1982), (Nikulin, 1992a, 1992b).

In our research (Lemeshko and Postovalov, 1998, 2001a, 2001b, 2002), (Lemeshko and Maklakov, 2004), (Lemeshko, 2004) statistic distributions of the nonparametric goodness-of-fit tests are investigated by the methods of statistical simulating and for constructed empirical distributions approximate models of law were founded. Obtained results were used for developing of recommendations for standardization (R 50.1.037-2002, 2002). Precise models of statistics distributions and the tables of upper percentage points presented in (Lemeshko and Lemeshko, 2009a, 2009b, 2009c, 2010). The comparative analysis results of power of goodness-of-fit tests in verification a single and composite hypothesis are presented in the paper (Lemeshko *et al.*, 2007, 2008, 2009).

3 DISTRIBUTIONS OF STATISTICS OF THE TESTS IN THE CASE OF VERIFICATION COMPOSITE HYPOTHESES CONCERNING GENERALIZED WEIBULL DISTRIBUTION

In the case of verification composite hypotheses concerning Generalized Weibull Distribution the distributions of statistics $G(S|H_0)$ for nonparametric goodness-of-fit tests depend on specific values of shape parameters θ_0.

In the fig. 3, fig. 4 you can see the behavior of statistics distribution S_Ω in testing composite hypotheses for family (1). In the case when three parameters are estimated by MLM (fig. 3) you can see the next: when the values of shape parameter are grow up till $\theta_1 \approx 2$ the distribution $G(S|H_0)$ is shift to the left. With the following growth of values of shape parameter the distribution $G(S|H_0)$ shifts to the opposite direction, to the right.

In the case when two parameters are estimated by MLM (fig. 4) you can see the following: with the growing of values of parameters θ_1 the distribution $G(S|H_0)$ shifts to the right.

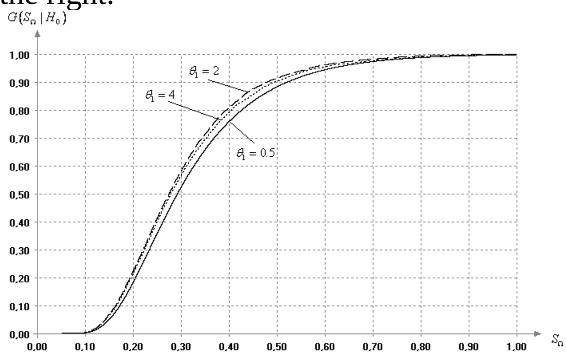

Fig. 3. Statistic distributions (5) of Anderson-Darling goodness-of-fit tests in composite hypotheses testing concerning family (1). MLM is used for estimation three parameters (θ_0, θ_1 and θ_2).

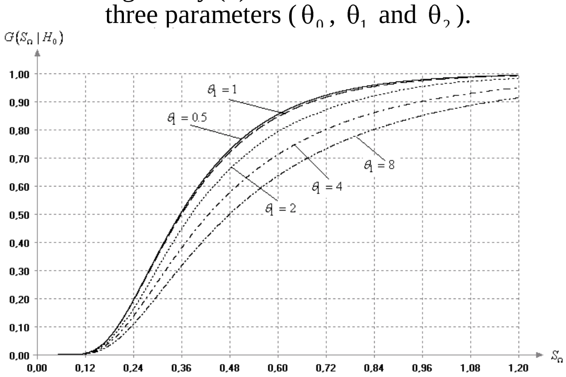

Fig. 4. Statistic distributions (5) of Anderson-Darling goodness-of-fit tests in composite hypotheses testing concerning family (1). MLM is used for estimation two (θ_0 and θ_1) parameters.

Percentage points obtained by statistic modeling and the models of marginal statistic distributions of Kolmogorov, Cramer-Von Mises-Smirnov and Anderson-Darling tests were computed for the values of shape parameter θ_1= 0.5, 1.0, 3.0, 4.0, 5.0 with MLM used for parameter estimation are presented in tables 1-5.

Distributions $G(S|H_0)$ of the Kolmogorov, Cramer-Von Mises-Smirnov and the Anderson-Darling statistics are best approximated by the family of the III type beta-distributions with the density function

$$B_3(\theta_0,\theta_1,\theta_2,\theta_3,\theta_4) =$$

$$= \frac{\theta_2^{\theta_0}}{\theta_3 \mathrm{B}(\theta_0,\theta_1)} \frac{\left(\frac{x-\theta_4}{\theta_3}\right)^{\theta_0-1}\left(1-\frac{x-\theta_4}{\theta_3}\right)^{\theta_1-1}}{\left[1+(\theta_2-1)\frac{x-\theta_4}{\theta_3}\right]^{\theta_0+\theta_1}},$$

or by the family of the *Sb*-Johnson distributions

$$Sb\ \theta_0,\theta_1,\theta_2,\theta_3\ =$$

$$= \frac{\theta_1\theta_2}{x-\theta_3\ \ \theta_2+\theta_3-x}\exp\left\{-\frac{1}{2}\left(\theta_0-\theta_1\ln\frac{x-\theta_3}{\theta_2+\theta_3-x}\right)^2\right\}.$$

The tables of percentage points and statistic distributions models were constructed by modeled statistic samples with the size $N=10^6$. In the case when $N=10^6$ the deviation the empirical p.d.f. $G_N(S|H_0)$ from the theoretical one is less than 10^{-3}. In this case the values of statistics of goodness-of-fit- tests were calculated using a samples of pseudorandom variables which belong to $F(x,\theta)$ with sample size $n=10^3$. For the such value of n statistic p.d.f. $G(S_n|H_0)$ almost coincides with the marginal p.d.f. $G(S|H_0)$.

4 CONCLUSIONS

The Generalized Weibull probability distribution plays an important role in a statistical analysis of lifetime or response data in reliability and survival studies. In certain parameters the function of Generalized Weibull Distribution agree with the Weibull distribution function and the exponential distribution function. In this work you can find how density of distribution depends on the parameters values of the law.

In this work are presented models of the statistic distributions of the nonparametric goodness-of-fit tests for testing composite hypotheses with the distributions family (1).

Table 1. Percentage points and models of limiting statistic distributions of the nonparametric goodness-of-fit test when MLM is used for parameter estimation (for θ_1 =0.5)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov's test		
θ_0	1.001	1.102	1.309	B_3 (6.5294, 6.8315, 3.5901, 2.0446,0.2801)
θ_1	1.084	1.199	1.427	B_3 (5.4860, 5.9744, 3.4348, 2.1402, 0.3000)
θ_2	1.038	1.144	1.360	B_3 (4.7833, 6.1285, 3.0596, 2.0214, 0.3200)
θ_0 , θ_1	0.849	0.922	1.071	B_3 (6.2332, 6.0259,2.8200, 1.3000, 0.2800)
θ_0 , θ_2	0.837	0.909	1.054	*Sb*(2.1787, 1.8756, 1.5259, 0.2567)
θ_1 , θ_2	0.848	0.922	1.076	*Sb*(2.4861, 1.8758, 1.7026, 0.2664)
θ_0 , θ_1 , θ_2	0.780	0.845	0.979	*Sb*(2.3507, 1.9291, 1.4629, 0.2495)
		for Cramer-Von Mises-Smirnov's test		
θ_0	0.181	0.232	0.359	B_3 (5.1297, 2.5959, 22.9591,0.8000, 0.0081)
θ_1	0.227	0.296	0.466	B_3 (7.4650, 2.6576, 44.4162, 1.3633, 0.0000)
θ_2	0.198	0.255	0.395	B_3 (5.4489, 2.7019, 31.5609, 1.1500, 0.0062)
θ_0 , θ_1	0.110	0.135	0.192	B_3 (6.3779, 4.6451, 27.3376, 1.0000, 0.0050)
θ_0 , θ_2	0.106	0.129	0.183	*Sb*(3.7541, 1.5434, 0.5800, 0.0058)
θ_1 , θ_2	0.112	0.138	0.200	B_3 (10.3369, 4.0734, 25.8270, 0.5802, 0.0000)
θ_0 , θ_1 , θ_2	0.086	0.103	0.145	B_3 (6.7252, 4.6508, 16.7920, 0.4800, 0.0050)
		for Anderson-Darling's test		
θ_0	1.125	1.415	2.140	B_3 (4.9800, 4.1685, 17.0454, 7.1000, 0.0500)
θ_1	1.279	1.625	2.478	B_3 (4.7602, 5.1000, 9.8527, 6.8675, 0.0000)
θ_2	1.157	1.454	2.186	B_3 (3.0331, 4.0598, 9.3429, 5.9880, 0.1000)
θ_0 , θ_1	0.673	0.806	1.120	B_3 (5.7172, 5.0419, 10.1641, 3.0044, 0.0550)
θ_0 , θ_2	0.655	0.781	1.079	*Sb*(3.8953, 1.6481, 3.5052, 0.0513)
θ_1 , θ_2	0.743	0.902	1.290	*Sb*(4.1462, 1.6136, 4.6254, 0.0535)
θ_0 , θ_1 , θ_2	0.523	0.617	0.839	*Sb*(3.9313, 1.6905, 2.7078, 0.0530)

Table 2. Percentage points and models of limiting statistic distributions of the nonparametric goodness-of-fit test when MLM is used for parameter estimation (for θ_1 = 1)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov's test		
θ_0	1.181	1.316	1.585	B_3 (6.9734, 4.8247, 5.3213, 2.3800, 0.2690)
θ_1	1.083	1.196	1.425	B_3 (4.6425, 6.6688, 2.8491, 2.2246, 0.3200)
θ_2	0.994	1.092	1.290	B_3 (6.2635, 7.1481, 3.2059, 2.0000, 0.2800)
θ_0 , θ_1	0.874	0.954	1.117	*Sb*(2.4299, 1.8866, 1.7504, 0.2598)
θ_0 , θ_2	0.823	0.893	1.033	B_3 (5.8989, 7.5040, 2.4180, 1.3724, 0.2800)
θ_1 , θ_2	0.815	0.883	1.023	*Sb*(2.4499, 1.9720, 1.6016, 0.2486)
θ_0 , θ_1 , θ_2	0.758	0.820	0.946	*Sb*(2.3012, 1.9386, 1.3863, 0.2464)
		for Cramer-Von Mises-Smirnov's test		
θ_0	0.320	0.431	0.706	B_3 (2.2422, 2.2970, 16.4663, 1.6500, 0.0130)
θ_1	0.227	0.295	0.464	B_3 (5.3830, 2.6954, 40.5199, 1.6450, 0.0050)
θ_2	0.174	0.221	0.336	B_3 (3.6505, 3.2499, 16.5445, 1.0000, 0.0100)
θ_0 , θ_1	0.117	0.144	0.209	*Sb*(3.8667, 1.4603, 0.7583, 0.0059)
θ_0 , θ_2	0.102	0.123	0.174	B_3 (12.2776, 4.1107, 27.2069, 0.4875, 0.0000)
θ_1 , θ_2	0.103	0.127	0.182	B_3 (4.7144, 4.6690, 10.8816, 0.5261, 0.0059)
θ_0 , θ_1 , θ_2	0.080	0.097	0.135	*Sb*(4.1842, 1.6587, 0.4794, 0.0061)

for Anderson-Darling's test				
θ_0	1.724	2.280	3.639	B_3 (4.8106, 2.6855, 35.5593,11.8700, 0.0500)
θ_1	1.275	1.617	2.468	B_3 (3.6999, 3.9108, 16.4841, 9.0300, 0.0740)
θ_2	1.056	1.314	1.953	B_3 (4.9871, 4.1479, 16.5432, 6.4500, 0.0600)
θ_0, θ_1	0.687	0.827	1.161	B_3 (4.6368, 6.6727, 7.1680, 3.6356, 0.0521)
θ_0, θ_2	0.633	0.753	1.037	B_3 (3.0467, 5.9239, 5.0944, 2.7870, 0.1000)
θ_1, θ_2	0.696	0.842	1.194	B_3 (6.9638, 4.5238, 17.7792, 3.8000, 0.0522)
θ_0, θ_1, θ_2	0.494	0.582	0.786	*Sb*(3.9578, 1.6861, 2.5760, 0.0547)

Table 3. Percentage points and models of limiting statistic distributions of the nonparametric goodness-of-fit test when MLM is used for parameter estimation (for θ_1 = 3)

Parameter estimated	Percentage points			Model
	0.9	0.95	0.99	
for Kolmogorov's test				
θ_0	1.150	1.279	1.538	B_3 (5.0155, 5.4869, 3.3992, 2.2476, 0.3000)
θ_1	1.084	1.199	1.427	B_3 (5.4860, 5.9744, 3.4348, 2.1402,0.3000)
θ_2	0.892	0.968	1.121	B_3 (4.6527, 7.8624, 1.8636, 1.4770, 0.3110)
θ_0, θ_1	0.992	1.095	1.301	B_3 (37.6836, 9.6249, 24.7703, 4.2400, 0.1000)
θ_0, θ_2	0.823	0.895	1.041	B_3 (6.6694, 6.5961, 3.0264,1.3700, 0.2650)
θ_1, θ_2	0.807	0.877	1.020	B_3 (5.3859, 8.4947, 2.3199, 1.4900, 0.2850)
θ_0, θ_1, θ_2	0.751	0.811	0.932	B_3 (5.7236, 7.0743, 2.3212, 1.1488, 0.2714)
for Cramer-Von Mises-Smirnov's test				
θ_0	0.286	0.383	0.620	*Sb*(3.4745, 1.1215, 2.1611, 0.0065)
θ_1	0.227	0.296	0.466	B_3 (8.0420, 2.6222, 50.1417, 1.3950, 0.0000)
θ_2	0.135	0.167	0.240	B_3 (9.8988, 3.6331, 27.2342, 0.6611, 0.0000)
θ_0, θ_1	0.167	0.215	0.334	*Sb*(3.6343, 1.2549, 1.1752, 0.0074)
θ_0, θ_2	0.100	0.121	0.172	B_3 (4.9109, 4.8805, 11.3991, 0.5400, 0.0058)
θ_1, θ_2	0.099	0.121	0.174	B_3 (9.7955, 5.0455, 35.0176, 0.9000, 0.0000)
θ_0, θ_1, θ_2	0.078	0.094	0.133	B_3 (4.2414, 3.7719, 8.6839, 0.2744, 0.0087)
for Anderson-Darling's test				
θ_0	1.556	2.040	3.234	B_3 (4.3943, 2.4670, 38.0035, 10.7000, 0.0900)
θ_1	1.279	1.625	2.478	B_3 (5.3689, 3.2667, 21.3222, 6.8675, 0.0535)
θ_2	0.900	1.096	1.572	B_3 (3.5132, 4.3501, 8.8168, 4.2500, 0.1000)
θ_0, θ_1	0.871	1.088	1.623	B_3 (5.6254, 3.7452, 20.0868, 4.9237, 0.0588)
θ_0, θ_2	0.619	0.737	1.015	B_3 (7.1939, 6.8828, 3.2613, 1.5626, 0.2598)
θ_1, θ_2	0.640	0.769	1.072	B_3 (30.1793 ,4.4373, 60.5986, 3.2000, 0.0000)
θ_0, θ_1, θ_2	0.483	0.568	0.773	B_3 (5.2772, 4.4958, 7.9102, 1.5891, 0.0664))

Table 4. Percentage points and models of limiting statistic distributions of the nonparametric goodness-of-fit test when MLM is used for parameter estimation (for θ_1 = 4)

Parameter estimated	Percentage points			Model
	0.9	0.95	0.99	
for Kolmogorov's test				
θ_0	1.131	1.256	1.506	B_3 (5.0752, 5.5757, 3.3089, 2.1797, 0.3000)
θ_1	1.084	1.199	1.427	B_3 (3.8892, 6.2974, 2.5413, 2.1402, 0.3400)
θ_2	0.890	0.966	1.119	*Sb*(2.1569,1.8555, 1.6361, 0.2661)
θ_0, θ_1	1.024	1.133	1.352	B_3 (14.6423, 5.3789, 9.0355, 2.1287, 0.2000)
θ_0, θ_2	0.839	0.914	1.068	B_3 (5.1515, 6.1071, 2.8573, 1.3900, 0.3000)
θ_1, θ_2	0.833	0.909	1.065	B_3 (7.3590, 7.0743, 3.0755, 1.4500, 0.2450)

$\theta_0, \theta_1, \theta_2$	0.769	0.834	0.970	B_3 (4.0431, 7.9330, 1.6664, 1.2059, 0.3007)
		for Cramer-Von Mises-Smirnov's test		
θ_0	0.269	0.359	0.578	*Sb*(3.4774, 1.1443, 1.9761, 0.0066)
θ_1	0.227	0.296	0.466	B_3 (7.2936, 2.6369, 40.7763, 1.2800, 0.0000)
θ_2	0.135	0.166	0.240	B_3 (6.9544, 4.2952, 17.0098, 0.7100, 0.0000)
θ_0, θ_1	0.186	0.242	0.379	B_3 (10.0457, 2.7234, 74.1688, 1.4000, 0.0000)
θ_0, θ_2	0.104	0.127	0.182	B_3 (10.3993, 4.2771, 25.5455, 0.5600, 0.0000)
θ_1, θ_2	0.104	0.128	0.186	B_3 (5.2006, 4.4814, 13.7165, 0.5770, 0.0050)
$\theta_0, \theta_1, \theta_2$	0.082	0.099	0.141	B_3 (4.3747, 3.2066, 9.2236, 0.2479, 0.0088)
		for Anderson-Darling's test		
θ_0	1.484	1.934	3.047	B_3 (12.5725, 2.7914, 75.0000, 9.6500, 0.0000)
θ_1	1.279	1.625	2.478	B_3 (6.9691, 2.9121, 32.3978, 6.8675, 0.0535)
θ_2	0.895	1.090	1.556	B_3 (16.0792, 4.1280, 41.0115, 4.9000, 0.0000)
θ_0, θ_1	0.958	1.213	1.838	B_3 (5.9821, 3.4306, 23.7037, 5.4000, 0.0500)
θ_0, θ_2	0.632	0.754	1.043	B_3 (19.4692, 4.7303, 32.4566, 2.8950, 0.0000)
θ_1, θ_2	0.645	0.776	1.087	B_3 (19.2831, 4.8148, 37.5002, 3.4100, 0.0000)
$\theta_0, \theta_1, \theta_2$	0.496	0.587	0.805	B_3 (5.9771,4.3144,9.7987,1.7085,0.0619)

Table 5. Percentage points and models of limiting statistic distributions of the nonparametric goodness-of-fit test when MLM is used for parameter estimation (for $\theta_1 = 5$)

Parameter estimated	Percentage points			Model
	0.9	0.95	0.99	
		for Kolmogorov's test		
θ_0	1.118	1.241	1.486	B_3 (4.7261, 5.4585, 3.2780, 2.1396, 0.3160)
θ_1	1.084	1.199	1.427	B_3 (4.0737, 5.8618, 2.8588, 2.1180, 0.3400)
θ_2	0.907	0.989	1.153	*Sb*(2.2461, 1.8567, 1.7288, 0.2655)
θ_0, θ_1	1.045	1.157	1.381	B_3 (5.7499, 6.4880, 3.3727, 2.1574, 0.2750)
θ_0, θ_2	0.858	0.937	1.098	B_3 (4.7054, 3.4609, 2.8365, 0.9900, 0.3000)
θ_1, θ_2	0.859	0.940	1.105	*Sb*(2.4735, 1.8966, 1.7594, 0.2472)
$\theta_0, \theta_1, \theta_2$	0.794	0.862	1.012	B_3 (6.8107,5.1464,3.5600,1.1675,0.2650)
		for Cramer-Von Mises-Smirnov's test		
θ_0	0.256	0.342	0.550	B_3 (3.9503, 2.5327, 30.3512, 1.6890, 0.0066)
θ_1	0.227	0.296	0.466	B_3 (7.5116, 2.6526,44.4963, 1.3550, 0.0000)
θ_2	0.140	0.173	0.252	*Sb*(2.2461, 1.8567, 1.7288, 0.2655)
θ_0, θ_1	0.199	0.260	0.409	B_3 (6.5731, 2.8621, 38.4570, 1.2800, 0.0000)
θ_0, θ_2	0.109	0.134	0.194	B_3 (9.3325, 3.9118, 20.9934, 0.4950, 0.0000)
θ_1, θ_2	0.110	0.137	0.201	B_3 (10.0324, 4.0962, 31.9612, 0.7050, 0.0000)
$\theta_0, \theta_1, \theta_2$	0.087	0.106	0.152	B_3 (4.3111, 4.0100, 12.1625, 0.4264, 0.0084)
		For Anderson-Darling's test		
θ_0	1.436	1.862	2.924	B_3 (4.8285, 3.2562, 24.8506, 9.4000, 0.0500)
θ_1	1.279	1.625	2.478	B_3 (4.7132, 3.5578, 16.7301, 6.8675, 0.0535)
θ_2	0.913	1.113	1.592	B_3 (14.4674, 4.1995, 38.4078, 5.2500, 0.0000)
θ_0, θ_1	1.025	1.305	1.995	B_3 (2.9220, 3.4603, 12.4402, 5.6400, 0.1000)
θ_0, θ_2	0.650	0.778	1.083	B_3 (18.0147, 4.7011, 31.4251, 3.0541, 0.0000)
θ_1, θ_2	0.660	0.796	1.126	B_3 (4.2178, 5.1030, 9.9078, 3.5340, 0.0800)
$\theta_0, \theta_1, \theta_2$	0.516	0.614	0.852	B_3 (5.0163, 4.7355, 9.8990, 2.2172, 0.0687)

It should be stressed, that obtained percentage points and models guarantee proper implementation of the nonparametric goodness-of-fit tests in statistic analysis problems if MLM is used. These results can't be used with other estimations because statistic distributions of these tests are essential depend on estimation method (Lemeshko *et al*., 2001b).

The authors hope that release of the article will be conductive to decrease mistake amount, committed in statistic analysis problems if nonparametric goodness-of-fit tests are used (Lemeshko, 2004).

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REFERENCES

Anderson T.W., and Darling D.A. 1952. Asymptotic theory of certain "goodness of fit" criteria based on stochastic processes. *Ann.Math. Statist*., 23: 193-212.

Anderson T.W., and Darling D.A. 1954. A test of goodness of fit. *J. Amer. Statist. Assoc*., 29: 765-769.

Bolshev L.N. 1987. On the question on testing some composite statistical hypotheses. In *Theory of Probability and Mathematical Statistics*. Selected Works. Nauka, Moscow, 5-63.

Bolshev L.N., Smirnov N.V. 1983. Tables of Mathematical Statistics. *Moscow: Science*. (in Russian)

Chandra M., Singpurwalla N.D., Stephens M.A. 1981. Statistics for Test of Fit for the Extrem–Value and Weibull Distribution. *J. Am. Statist. Assoc*. 76(375): 729-731.

Darling D.A. 1955. The Cramer-Smirnov test in the parametric case. *Ann. Math. Statist*., 26: 1-20.

Darling D.A. 1957. The Cramer-Smirnov test in the parametric case. *Ann. Math. Statist*., 28: 823-838.

Durbin J. 1973. Weak convergence of the sample distribution function when parameters are estimated. *Ann. Statist*., 1: 279-290.

Durbin J. 1975. Kolmogorov-Smirnov tests when parameters are estimated with applications to tests of exponentiality and tests of spacings. *Biometrika*, 62: 5-22.

Durbin J. 1976. Kolmogorov–Smirnov Test when Parameters are Estimated. *Lect. Notes Math*. 566: 33–44.

Dzhaparidze K.O., and Nikulin M.S. 1982. Probability distribution of the Kolmogorov and omega-square statistics for continuous distributions with shift and scale parameters. *J. Soviet Math*., 20: 2147-2163.

Gihman I.I. 1953. Some remarks on the consistency criterion of A.N. Kolmogorov. *Dokl. Akad. Nauk SSSR*, 91(4): 715-718.

Gihman I.I. 1961. On the empirical distribution function in the case of grouping data. In *Selected Translations in Mathematical Statistics and Probability*, vol. 1. American Math. Soc., Providence, RI, 77-81.

Kac M., Kiefer J., Wolfowitz J. 1955. On Tests of Normality and Other Tests of Goodness of Fit Based on Distance Methods. *Ann. Math. Stat*. 26: 189-211.

Lemeshko B.Yu. 2004. Errors when using nonparametric fitting criteria. *Measurement Techniques*, 47(2): 134-142.

Lemeshko B.Yu., Postovalov S.N. 1998. Statistical Distributions of Nonparametric Goodness-of-Fit Tests as Estimated by the Sample Parameters of Experimentally Observed Laws. *Industrial laboratory*, 64(3): 197-208.

Lemeshko B.Yu., Postovalov S.N. 2001a. Application of the Nonparametric Goodness-of-Fit Tests in Testing Composite Hypotheses. *Optoelectronics, Instrumentation and Data Processing* 37(2): 76-88.

Lemeshko B.Yu., Postovalov S.N. 2001b. About dependence of statistics distributions of the nonparametric tests and their power from a method of the parameters estimation. *Industrial laboratory. Diagnostics of materials*, 67(7): 62-71. (in Russian)

Lemeshko B.Yu., Postovalov S.N. 2002. The nonparametric goodness-of-fit tests about fit with Johnson distributions in testing composite hypotheses. *News of the SB AS HS*, 1(5): 65-74. (in Russian)

Lemeshko B. Yu. 2004. Errors when using nonparametric fitting criteria. *Measurement Techniques*, 47(2): 134-142.

Lemeshko B.Yu., Maklakov A.A. 2004. Nonparametric Test in Testing Composite Hypotheses on Goodness of Fit Exponential Family Distributions. *Optoelectronics, Instrumentation and Data Processing*, 40(3): 3-18.

Lemeshko B.Yu., Lemeshko S.B., Postovalov S.N. 2007. The power of goodness-of-fit tests for close alternatives. *Measurement Techniques*, 50(2): 132-141.

Lemeshko B.Yu., Lemeshko S.B., Postovalov S.N. 2009. Comparative Analysis of the Power of Goodness-of-Fit Tests for Near Competing Hypotheses. I. The Verification of Simple Hypotheses. *Journal of Applied and Industrial Mathematics*, **3**(4): 462–475.

Lemeshko B.Yu. Lemeshko S.B., Postovalov S.N., 2010. Comparative analysis of the power of goodness-of-fit tests for near competing hypotheses. II. Verification of complex hypotheses, *Journal of Applied and Industrial Mathematics*, **4(**1), 79–93.

Lemeshko S.B., Lemeshko B.Yu. 2007. Statistic distributions of the nonparametric goodness-of-fit tests in testing hypotheses relative to beta-distributions. *News of the SB AS HS*, 2(9): 6-16. (in Russian)

Lemeshko B.Yu., Lemeshko S.B. 2009a. Distribution models for nonparametric tests for fit in verifying complicated hypotheses and maximum-likelihood estimators. Part 1. *Measurement Techniques*. **52**(6): 555-565.

Lemeshko B.Yu., Lemeshko S.B. 2009b. Models for statistical distributions in nonparametric fitting tests on composite hypotheses based on maximum-likelihood estimators. Part II. *Measurement Techniques*, **52**(8): 799-812.

Lemeshko B.Yu., Lemeshko S.B. 2009c. Models of Statistic Distributions of Nonparametric Goodness-of-fit Tests in Composite Hypotheses Testing in Case of Double Exponential Law. *The XIII International Conference "Applied Stochastic Models and Data Analysis"* (ASMDA-2009), June 30-Jule 3, Selected papers. Vilnius, Lithania: 153-157.

Lemeshko B. Yu., Lemeshko S. B., Postovalov S.N. 2010. Statistic Distribution Models for some nonparametric goodness-of-fit tests in testing composite hypothesis, *Communications in Statistics - Theory and Methods*. **39**(3): 460-471.

Lemeshko S.B. 2007. Expansion of applied opportunities of some classical methods of mathematical statistics. *The dissertation on competition of a scientific degree of Cand. Tech. Sci*. Novosibirsk State Technical University. Novosibirsk. (in Russian)

Martinov G.V. 1978. Omega-Quadrate Tests. *Moscow: Science*. (in Russian)

Nikulin M.S. 1992a. Gihman and goodness-of-fit tests for grouped data. *Mathematical Reports of the academy of Science of the Royal Society of Canada*, 14(4): 151-156.

Nikulin M.S. 1992b. A variant of the generalized omega-square statistic. *J. Soviet Math*., 61 (4):1896-1900, (translation from Zapiski nauchnikh seminarov LOMI, (1989), 177: 108-113).

Pearson E.S., Hartley H.O. 1972. Biometrica Tables for Statistics. Vol. 2. Cambridge: University Press.

R 50.1.037-2002. 2002. Recommendations for Standardization. Applied statistics. Rules of check of experimental and theoretical distribution of the consent. Part II. Nonparametric goodness–of–fit test. *Moscow: Publishing house of the standards*. (in Russian)

Stephens M.A. 1970. Use of Kolmogorov–Smirnov, Cramer – von Mises and Related Statistics – Without Extensive Table. *J. R. Stat. Soc*., 32: 115-122.

Stephens M.A. 1974. EDF Statistics for Goodness of Fit and Some Comparisons. *J. Am. Statist. Assoc*., 69: 730-737.

Tyurin Yu.N. 1984. On the Limiting Kolmogorov-Smirnov Statistic Distribution for Composite Hypothesis. *News of the AS USSR. Ser. Math*., 48(6): 1314-1343. (in Russian)

Tyurin Yu.N., Savvushkina N.E. 1984. Goodness-of-Fit Test for Weibull-Gnedenko Distribution. *News of the AS USSR. Ser. Techn. Cybernetics*, 3: 109-112. (in Russian)