Classical tests of variances homogeneity for non-normal distributions

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ABSTRACT: The comparative analysis of power of classical variance homogeneity tests (Fisher's, Bartlett's, Cochran's, Hartley's and Levene's tests) is carried out. Distributions of tests statistics are investigated under violation of assumptions that samples belong to the normal law. Distributions and power of nonparametric tests of homogeneity of dispersion characteristics are researched (Ansari-Bradley's, Mood's, Siegel-Tukey's tests). The comparative analysis of power of classical variance homogeneity tests with power of nonparametric tests is carried out. Tables of percentage points for Cochran's test are presented in case of the distributions which are different from normal.

1 INTRODUCTION

Tests of samples homogeneity are often used in various applications of statistical analysis. The question can be about checking hypotheses about homogeneity of samples distributions, population means or variances. Naturally the most complete findings can be done in the first case. However researcher can be interested in possible deviations in the sample mean values or differences in dispersion characteristics of measurements results.

Application features of nonparametric Smirnov and Lehmann-Rosenblatt homogeneity tests and analysis of their power were considered in (Lemeshko & Lemeshko (2005)). In (Lemeshko & Lemeshko (2008)) it was shown that classical criteria for testing hypotheses about homogeneity of means are stable to violation of normality assumption and comparative analysis of the power of various tests, including nonparametric, was given.

One of the basic assumptions in constructing classical tests for equality of variances is normal distribution of observable random variables (measurement errors). Therefore the application of classical criteria always involves the question of how valid the results obtained are in this particular situation. Under violation of assumption that analyzed variables belong to normal law, conditional distributions of tests statistics, when hypothesis under test is true, change appreciably.

All available publications do not give full information on the power of the classical tests for homogeneity of variances and on comparative analysis of the power of the classical tests and nonparametric criteria for testing hypotheses about the homogeneity of the dispersion characteristics (scale parameters).

This work continues researches of stability of criteria for testing hypotheses about the equality of variances (Lemeshko & Mirkin (2004)). Classical Bartlett's (Bartlett (1937)), Cochran's (Cochran (1941)), Fisher's, Hartley's (Hartley (1950)), Levene's (Levene (1960)) tests have been compared, nonparametric (rank) Ansari-Bradley's (Ansari & Bradley (1960)), Mood's (Mood (1954)), Siegel-Tukey's (Siegel & Tukey (1960)) tests have been considered. The purpose of the paper is

– research of statistics distributions for listed tests in case of distribution laws of observable random variables which are different from normal;
– comparative analysis of criteria power concerning concrete competing hypotheses;
– realization of the possibility to apply the classical tests under violation of assump-

tions about normality of random variables.

A hypothesis under test for equality of variances corresponding to m samples will have the form

$$H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_m^2, \quad (1)$$

and the competitive hypothesis is

$$H_1 : \sigma_{i_1}^2 \neq \sigma_{i_2}^2, \quad (2)$$

where the inequality holds at least for one pair of subscripts i_1, i_2.

Statistical simulation methods and the developed software have been used for investigating statistic distributions, calculating percentage points and estimating tests power with respect to various competing hypotheses. The sample size of statistics under study was $N = 10^6$. Such N allowed absolute value of difference between true law of statistics distribution and simulated empirical not to exceed 10^{-3}.

Statistic distributions have been studied for various distribution laws, in particular, in case when simulated samples belong to the family with density

$$De(\theta_0) = f(x;\theta_0,\theta_1,\theta_2) = \frac{\theta_0}{2\theta_1 \Gamma(1/\theta_0)} \exp\left(-\left(\frac{|x-\theta_2|}{\theta_1} \right)^{\theta_0} \right) \quad (3)$$

with various values of the form parameter θ_0. This family can be a good model for error distributions of various measuring systems. Special cases of distribution $De(\theta_0)$ include the Laplace $(\theta_0 = 1)$ and normal $(\theta_0 = 2)$ distribution. The family (3) allows to define various symmetric distributions that differ from normal: the smaller value of form parameter θ_0 the "heavier" tails of the distribution $De(\theta_0)$, and vice-versa the higher value the "easier" tails.

The competing hypotheses of the form $H_1 : \sigma_m = d\sigma_0$ have been considered in comparative analysis of the test power. That is, a competing hypothesis corresponds to the situation when $m-1$ samples belong to the law with $\sigma = \sigma_0$, while one of the samples, for example, with number m has some different variance. Hypothesis under test corresponds to the situation

$$H_0 : \sigma_1^2 = \sigma_2^2 = ... = \sigma_m^2 = \sigma_0^2 .$$

2 CLASSICAL TESTS OF VARIANCES HOMOGENEITY

2.1 *Bartlett's test*

Bartlett's test statistic (Bartlett (1937)) is

$$B = M \left[1 + \frac{1}{3(m-1)} \left(\sum_{i=1}^{m} \frac{1}{\nu_i} - \frac{1}{N} \right) \right]^{-1} \quad (4)$$

where

$$M = N \ln\left(\frac{1}{N} \sum_{i=1}^{m} \nu_i S_i^2 \right) - \sum_{i=1}^{m} \nu_i \ln S_i^2 ,$$

m is the number of samples; n_i are the sample sizes; $\nu_i = n_i$, if mathematical expectation is known, and $\nu_i = n_i - 1$, if it is unknown;

$$N = \sum_{i=1}^{m} \nu_i ;$$

S_i^2 – estimators of the sample variances. If the mathematical expectation is unknown, the estimators are

$$S_i^2 = \frac{1}{n_i - 1} \sum_{j=1}^{n_i} (X_{ji} - \bar{X}_i)^2 ,$$

where X_{ij} – j-th observation in sample i,

$$\bar{X}_i = \frac{1}{n_i} \sum_{j=1}^{n_i} X_{ji} .$$

If hypothesis H_0 is true, all $\nu_i > 3$ and samples are extracted from a normal population, then the statistic (4) has approximately the χ_{m-1}^2 distribution. If measurements are normally distributed, the distribution for the statistic (4) is almost independent of the sample sizes n_i (Lemeshko & Mirkin (2004)). If distributions of observed variables differ from the normal law, the distribution $G(B \mid H_0)$ of statistic (4) becomes depending on n_i and differs from χ^2_{m-1}.

2.2 *Cochran's test*

When all n_i are equal, one can use simpler Cochran's test (Cochran (1941)). The test statistic Q is defined as follows:

$$Q = \frac{S^2_{\max}}{S_1^2 + S_2^2 + \ldots + S_m^2}, \tag{5}$$

where $S^2_{\max} = \max(S_1^2, S_2^2, \ldots, S_m^2)$, m is the number of independent estimators of variances (number of samples), S_i^2 are estimators of the sample variances.

Distribution of Cochran's test statistic strongly depends on the sample size. The reference literature gives only tables of the percentage points for limited number of values n, which are used in hypothesis testing.

2.3 *Hartley's test*

Hartley's test (Hartley (1950)) as well as Cochran's test is used in case of samples of equal size.

Hartley's test statistic for homogeneity of variances is

$$H = \frac{S^2_{\max}}{S^2_{\min}}, \tag{6}$$

where

$S^2_{\max} = \max(S_1^2, S_2^2, \ldots, S_m^2)$, $S^2_{\min} = \min(S_1^2, S_2^2, \ldots, S_m^2)$, m – number of independent estimators of variances (number of samples).

Literature gives tables of percentage points for distribution of statistic (6) depending on $\nu_i = m$ and $\nu_2 = n-1$.

2.4 *Levene's test*

The Levene's test statistic (Levene (1960)) is defined as:

$$W = \frac{N-m}{m-1} \frac{\sum_{i=1}^{m} n_i (\bar{Z}_{i\bullet} - \bar{Z}_{\bullet\bullet})^2}{\sum_{i=1}^{m} \sum_{j=1}^{n_i} (Z_{ij} - \bar{Z}_{i\bullet})^2}, \tag{7}$$

where m is the number of samples, n_i is the sample size of the i-th sample,

$$N = \sum_{i=1}^{m} n_i,$$

$Z_{ij} = \left| X_{ij} - \bar{X}_{i\bullet} \right|$, X_{ij} – j-th observation in sample i, $\bar{X}_i$ is the mean of i-th sample, $\bar{Z}_{i\bullet}$ is the mean of the Z_{ij} for sample i, $\bar{Z}_{\bullet\bullet}$ – the mean of all Z_{ij}.

In some descriptions of the test, it is said that in case when samples belong to the normal law and hypothesis H_0 is true, the statistic has a F_{ν_1,ν_2} - distribution with number of degrees of freedom $\nu_1 = m-1$ and $\nu_2 = N-m$. Actually *distribution of statistics (7) is not Fisher's distribution* F_{ν_1,ν_2}. Therefore percentage points of distribution were investigated using statistical simulation methods (Neel & Stallings (1974)).

Levene's test is less sensitive to departures from normality. However it has less power.

The original Levene's test used only sample means. Brown and Forsythe (Brownl & Forsythe (1974)) suggested using sample median and trimmed mean as estimators of the mean for statistic (7).

However our researches have shown that *using in (7) sample median and trimmed mean leads to another distribution* $G(W \mid H_0)$ *of statistics (7).*

2.5 *Fisher's test*

Fisher's test is used to check hypothesis of variances homogeneity for *two* samples of random variables. The test statistic has a simple form

$$F = \frac{s_1^2}{s_2^2}, \tag{8}$$

where s_1^2 and s_2^2 – unbiased variance estimators, computed from the sample data.

In case when samples belong to the normal law and hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$ is true, this statistic has the F_{ν_1,ν_2} -distribution with number of degrees of freedom $\nu_1 = n_1 - 1$ and $\nu_1 = n_2 - 1$. A hypothesis under test is rejected if $F^* < F_{\alpha/2,\nu_1,\nu_2}$ or $F^* < F_{1-\alpha/2,\nu_1,\nu_2}$.

3 NONPARAMETRIC (RANK) TESTS

3.1 *Ansari-Bradley's test*

Nonparametric analogues of tests for homogeneity of variances are used to check hypothesis that two samples with sample sizes n_1 and n_2 belong to population with identical characteristics of dispersion. As a rule equality of means is supposed.

The Ansari-Bradley's test statistic (Ansari & Bradley (1960)) is:

$$S = \left\{ \sum_{i=1}^{n_1} \frac{n_1 + n_2 + 1}{2} - \left| R_i - \frac{n_1 + n_2 + 1}{2} \right| \right\} \qquad (9)$$

where R_i - ranks corresponding to elements of the first sample in general variational row.

In case when samples belong to the same law and checked hypothesis H_0 is true, distribution of statistics (9) does not depend on this law. *Discreteness* of distribution of statistics (9) *can be practically neglected* when $n_1, n_2 > 40$.

3.2 *Siegel-Tukey's test*

The variational row constructed on general sample $x_1 \le x_2 \le ... \le x_n$, where $n = n_1 + n_2$, is transformed into such sequence

$$x_1, x_n, x_{n-1}, x_2, x_3, x_{n-2}, x_{n-3}, x_4, x_5 ...,$$

i.e. row of remained values is "turned over" each time when ranks are assigned to pair of extreme values. Sum of ranks of sample with smaller size is used as test statistics. When $n_1 < n_2$ test statistic (Siegel & Tukey (1960)) is defined as:

$$R = \sum_{i=1}^{n_1} R_i \,, \qquad (10)$$

Discreteness of distribution of statistics (10) *can be practically neglected* when $n_1, n_2 > 30$.

3.3 *Mood's test*

The test statistic (Mood (1954)) is:

$$M = \sum_{i=1}^{n_1} \left(R_i - \frac{n_1 + n_2 + 1}{2} \right)^2 , \qquad (11)$$

where R_i - ranks of sample with smaller size in general variational row. *Discreteness* of distribution of statistics (11) *can be neglected at all* when $n_1, n_2 > 20$.

When sample sizes $n_1, n_2 > 10$ discrete distributions of statistics (9), (10) and (11) are well enough approximated by normal law. Therefore instead of statistics (9), (10) and (11) normalized analogues are more often used, which are approximately standard normal.

4 COMPARATIVE ANALYSIS OF POWER

At given probability of type I error α (to reject the null hypothesis when it is true) it is possible to judge advantages of the test by value of power $1-\beta$, where β is the probability of type II error (not to reject the null hypothesis when alternative is true). In (Bol'shev & Smirnov (1983)) it is definitely said that Cochran's test has lower power in comparison with Bartlett's test. In (Lemeshko & Mirkin (2004)) it was shown that Cochran's test has greater power by the example of checking hypothesis about variances homogeneity for *five* samples.

Research of power of Bartlett's, Cochran's, Hartley's, Fisher's and Levene's tests concerning such competing hypotheses $H_1 : \sigma_2 = d\sigma_1, d \ne 1$ (in case of two samples that belong to the normal law) has shown that Bartlett's, Cochran's, Hartley's and Fisher's tests have equal power in this case. Levene's test appreciably yields to them in power.

In case of the distributions which are different from normal, for example, family of distributions with density (3), Bartlett's, Cochran's, Hartley's and Fisher's tests remain equivalent in power, and Levene's test also appreciably yields to them. However in case of heavy-tailed distributions (for example, when samples belong to the Laplace distribution) Levene's test has advantage of greater power.

Bartlett's, Cochran's, Hartley's and Levene's tests can be applied when number of samples $m > 2$. In such situations power of these tests is different. If $m > 2$ and normality assumption is true, given tests can be ordered by power decrease as follows: *Cochran's* $\succ$ *Bartlett's* $\succ$ *Hartley's* $\succ$ *Levene's*.

The preference order remains in case of violation of normality assumption. The exception concerns situations when samples belong to laws with more “heavy tails” in comparison with the normal law. For example, in case of Laplace distribution Levene's test is more powerful than three others.

Results of nonparametric criteria power research have shown appreciable advantage of Mood's test and practical equivalence of Siegel-Tukey's and Ansari-Bradley's tests. Of course, nonparametric tests yield in power to Bartlett's, Cochran's, Hartley's and Fisher's tests. Figure 1 shows graphs of criteria power concerning competing hypotheses $H_1^1 : \sigma_2 = 1.1\sigma_1$ and $H_1^2 : \sigma_2 = 1.5\sigma_1$ depending on sample size n_i in case when $\alpha = 0.1$ and samples belong to the normal law. As we see, advantage in power of Cochran's test is rather significant in comparison with Mood's test - most powerful of nonparametric tests. Let's remind that Bartlett's, Cochran's, Hartley's and Fisher's tests have equal power in case of two samples.

Distributions of nonparametric tests statistics do not depend on a law kind, if both samples belong to the same population. But if samples belong to different laws and hypothesis of variances equality H_0 is true, *distributions of statistics of nonparametric tests depend on a kind of these laws.*

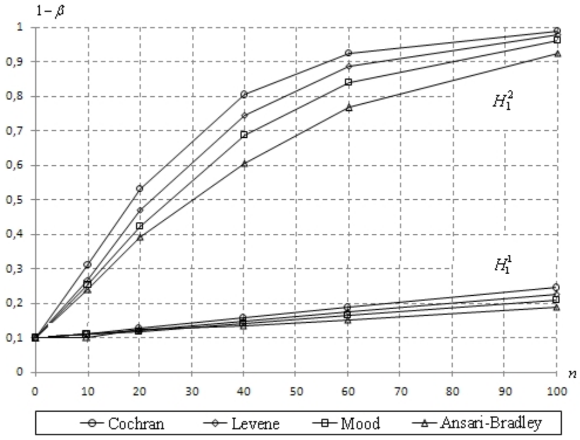

Figure 1. Power of tests concerning competing hypotheses H_1^1 and H_1^2 depending on sample size n when $\alpha = 0.1$ and samples belong to normal law.

5 COCHRAN'S TEST IN CASE OF LAWS DIFFERENT FROM NORMAL

Classical tests have considerable advantage in power over nonparametric. This advantage remains when analyzed samples belong to the laws appreciably different from normal. Therefore there is every reason to research statistics distributions of classical tests for checking variances homogeneity (construction of distributions models or tables of percentage points) in case of laws most often used in practice (different from the normal law). Among considered tests Cochran's test is the most suitable for this role.

In case when observable variables belong to family of distributions (3) with parameter of the form $\theta_0 = 1,2,3,4,5$ and some values n, tables 1-4 of upper percentage points (1%, 5%, 10%) for Cochran's test were obtained using statistical simulation (when number of samples $m = 2 \div 5$). The results obtained can be used in situations when distribution (3) with appropriate parameter θ_0 is a good model for observable random variables. Computed percentage points improve some results presented in (Lemeshko & Mirkin (2004)) and expand possibilities to apply Cochran's test.

Table 1. Upper percentage points for Cochran's test statistic distribution in case of 2 samples with equal size n

n	*De*(1) α			*De*(2) α			*De*(3) α			*De*(4) α			*De*(5) α		
	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
5	0.917	0.947	0.980	0.865	0.906	0.959	0.845	0.890	0.950	0.836	0.883	0.947	0.831	0.879	0.945
8	0.862	0.900	0.949	0.791	0.833	0.899	0.764	0.807	0.877	0.751	0.794	0.866	0.744	0.787	0.861
10	0.836	0.875	0.930	0.761	0.801	0.868	0.733	0.773	0.842	0.720	0.759	0.829	0.713	0.751	0.822
15	0.789	0.829	0.890	0.713	0.748	0.811	0.686	0.719	0.780	0.674	0.706	0.765	0.667	0.698	0.757
20	0.759	0.797	0.858	0.684	0.716	0.774	0.660	0.689	0.743	0.648	0.676	0.728	0.642	0.669	0.720
25	0.736	0.772	0.834	0.665	0.694	0.748	0.642	0.668	0.717	0.632	0.656	0.703	0.626	0.649	0.695
30	0.718	0.753	0.814	0.650	0.677	0.727	0.629	0.653	0.699	0.619	0.642	0.685	0.614	0.635	0.677
40	0.693	0.725	0.782	0.630	0.654	0.699	0.611	0.632	0.672	0.603	0.622	0.660	0.598	0.616	0.653
50	0.674	0.704	0.758	0.617	0.638	0.679	0.599	0.618	0.654	0.591	0.609	0.642	0.587	0.604	0.636
60	0.660	0.689	0.740	0.606	0.626	0.664	0.591	0.608	0.640	0.583	0.599	0.630	0.579	0.594	0.624
70	0.649	0.676	0.724	0.598	0.617	0.652	0.584	0.599	0.630	0.577	0.591	0.620	0.573	0.587	0.614
80	0.640	0.665	0.712	0.592	0.609	0.642	0.578	0.593	0.621	0.572	0.585	0.612	0.568	0.581	0.607
90	0.632	0.657	0.701	0.587	0.603	0.634	0.573	0.587	0.614	0.567	0.580	0.605	0.564	0.576	0.600
100	0.626	0.649	0.692	0.582	0.598	0.628	0.570	0.583	0.609	0.564	0.576	0.600	0.561	0.572	0.595

Table 2. Upper percentage points for Cochran's test statistic distribution in case of 3 samples with equal size n

n	*De*(1) α			*De*(2) α			*De*(3) α			*De*(4) α			*De*(5) α		
	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
5	0.794	0.847	0.918	0.700	0.752	0.839	0.665	0.717	0.806	0.649	0.700	0.790	0.641	0.690	0.781
8	0.716	0.768	0.852	0.614	0.658	0.741	0.579	0.620	0.698	0.563	0.602	0.677	0.554	0.591	0.665
10	0.681	0.732	0.817	0.581	0.622	0.698	0.548	0.584	0.654	0.533	0.567	0.634	0.524	0.557	0.622
15	0.623	0.669	0.751	0.531	0.564	0.628	0.503	0.531	0.588	0.489	0.516	0.569	0.482	0.508	0.558
20	0.587	0.629	0.707	0.502	0.531	0.588	0.477	0.501	0.550	0.466	0.488	0.533	0.459	0.480	0.524
25	0.562	0.600	0.673	0.484	0.509	0.560	0.461	0.482	0.526	0.450	0.470	0.510	0.444	0.463	0.501
30	0.543	0.578	0.647	0.470	0.493	0.539	0.449	0.468	0.507	0.439	0.457	0.493	0.434	0.451	0.485
40	0.515	0.547	0.608	0.450	0.470	0.510	0.432	0.449	0.482	0.424	0.439	0.470	0.419	0.434	0.463
50	0.496	0.525	0.581	0.437	0.455	0.490	0.421	0.436	0.465	0.414	0.427	0.454	0.410	0.422	0.448
60	0.482	0.508	0.560	0.428	0.444	0.476	0.413	0.426	0.453	0.406	0.418	0.443	0.402	0.414	0.437
70	0.471	0.495	0.543	0.421	0.435	0.465	0.407	0.419	0.444	0.401	0.412	0.434	0.397	0.408	0.429
80	0.462	0.485	0.530	0.415	0.429	0.456	0.402	0.413	0.436	0.396	0.406	0.427	0.393	0.403	0.422
90	0.455	0.476	0.518	0.410	0.423	0.449	0.398	0.408	0.430	0.392	0.402	0.422	0.389	0.398	0.417
100	0.449	0.469	0.509	0.406	0.418	0.443	0.394	0.405	0.425	0.389	0.398	0.417	0.386	0.395	0.413

Table 3. Upper percentage points for Cochran's test statistic distribution in case of 4 samples with equal size n

n	*De*(1) α 0.1	0.05	0.01	*De*(2) α 0.1	0.05	0.01	*De*(3) α 0.1	0.05	0.01	*De*(4) α 0.1	0.05	0.01	*De*(5) α 0.1	0.05	0.01
5	0.696	0.755	0.848	0.584	0.634	0.727	0.545	0.591	0.679	0.527	0.571	0.656	0.517	0.560	0.643
8	0.611	0.666	0.761	0.501	0.541	0.619	0.466	0.500	0.569	0.450	0.482	0.546	0.441	0.471	0.533
10	0.575	0.626	0.720	0.470	0.506	0.576	0.438	0.468	0.529	0.423	0.451	0.507	0.415	0.441	0.495
15	0.517	0.561	0.646	0.424	0.453	0.510	0.397	0.421	0.468	0.385	0.406	0.450	0.378	0.398	0.439
20	0.482	0.521	0.598	0.399	0.422	0.471	0.375	0.395	0.435	0.364	0.382	0.419	0.358	0.375	0.410
25	0.457	0.493	0.563	0.382	0.403	0.445	0.360	0.378	0.413	0.351	0.366	0.398	0.346	0.360	0.390
30	0.439	0.471	0.536	0.369	0.388	0.427	0.350	0.365	0.397	0.341	0.355	0.384	0.336	0.349	0.377
40	0.413	0.441	0.498	0.352	0.368	0.401	0.335	0.348	0.348	0.328	0.340	0.364	0.324	0.335	0.358
50	0.395	0.420	0.470	0.340	0.355	0.384	0.326	0.337	0.361	0.319	0.329	0.351	0.315	0.325	0.345
60	0.382	0.404	0.451	0.332	0.345	0.371	0.319	0.329	0.350	0.313	0.322	0.341	0.309	0.318	0.336
70	0.372	0.392	0.435	0.326	0.337	0.361	0.313	0.323	0.342	0.308	0.316	0.334	0.305	0.313	0.329
80	0.364	0.383	0.422	0.320	0.331	0.354	0.309	0.318	0.336	0.304	0.312	0.328	0.301	0.309	0.324
90	0.357	0.375	0.412	0.316	0.326	0.348	0.305	0.314	0.331	0.300	0.308	0.324	0.298	0.305	0.320
100	0.352	0.368	0.403	0.313	0.322	0.342	0.302	0.310	0.327	0.298	0.305	0.320	0.295	0.302	0.316

Table 4. Upper percentage points for Cochran's test statistic distribution in case of 5 samples with equal size n

n	*De*(1) α 0.1	0.05	0.01	*De*(2) α 0.1	0.05	0.01	*De*(3) α 0.1	0.05	0.01	*De*(4) α 0.1	0.05	0.01	*De*(5) α 0.1	0.05	0.01
5	0.623	0.684	0.787	0.504	0.551	0.642	0.464	0.505	0.588	0.446	0.484	0.562	0.436	0.472	0.548
8	0.537	0.591	0.690	0.426	0.461	0.533	0.392	0.421	0.482	0.376	0.403	0.458	0.367	0.393	0.446
10	0.501	0.550	0.645	0.397	0.428	0.491	0.366	0.392	0.444	0.352	0.375	0.422	0.344	0.366	0.411
15	0.445	0.485	0.567	0.355	0.379	0.429	0.330	0.349	0.390	0.318	0.336	0.372	0.312	0.329	0.363
20	0.412	0.447	0.520	0.332	0.352	0.394	0.310	0.326	0.360	0.300	0.315	0.345	0.295	0.308	0.337
25	0.388	0.420	0.485	0.316	0.334	0.371	0.297	0.311	0.341	0.288	0.301	0.328	0.283	0.295	0.320
30	0.370	0.399	0.459	0.305	0.321	0.354	0.287	0.300	0.327	0.279	0.291	0.315	0.275	0.286	0.308
40	0.347	0.371	0.371	0.290	0.303	0.331	0.275	0.285	0.308	0.268	0.278	0.298	0.264	0.273	0.292
50	0.330	0.352	0.397	0.280	0.291	0.316	0.266	0.276	0.295	0.260	0.269	0.286	0.257	0.265	0.281
60	0.318	0.337	0.378	0.272	0.283	0.304	0.220	0.227	0.242	0.254	0.262	0.278	0.252	0.259	0.274
70	0.309	0.326	0.363	0.266	0.276	0.296	0.255	0.263	0.279	0.250	0.257	0.272	0.247	0.254	0.268
80	0.301	0.318	0.352	0.262	0.271	0.289	0.251	0.259	0.274	0.247	0.253	0.267	0.244	0.250	0.263
90	0.295	0.310	0.342	0.258	0.266	0.284	0.248	0.255	0.269	0.244	0.250	0.263	0.242	0.247	0.259
100	0.290	0.304	0.334	0.255	0.263	0.279	0.246	0.252	0.265	0.242	0.247	0.259	0.239	0.245	0.256

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