The analytical review of tests for randomness and the absence of a trend

I. V. Veretel`nikova, B. Yu. Lemeshko
Novosibirsk State Technical University

Annotation – For a variety of parametric and nonparametric tests, designed to test hypotheses of randomness or absence of a trend the distribution of statistics were investigated with Monte-Carlo algorithm, corresponding to the truth of the hypothesis under test in accordance with sample sizes. Procedure of interactive simulation of statistic distributions tests is proposed and implemented that allowed to apply the relevant test correctly in conditions of violation of standard assumptions. The results of comparative analysis of power of the tests against competing hypotheses hypothesis with different models of linear trend, conclusions about preferability of using a particular test are made.

Key words **– hypothesis of randomness, trend, Monte-Carlo simulation, power of the test.**

I. INTRODUCTION

INDICATION OF THE PRESENCE in the observed random sequence of measurements of a certain non-random patterns may be deviation in the hypothesis under test of randomness or absence of a trend. Various parametric and nonparametric tests have been offered to test such hypothesis at different times. However, available sources do not allow us to judge the benefits of a particular test. Current works do not provide clear recommendations drawing around area of application and assumption, performance of which provides correctness of statistical conclusions when using considered test.

Fairly comprehensive list of tests aimed at verifying the hypothesis of randomness and absence of a trend is presented in work [1]. It can be considered as a reference that covers a sufficiently broad set of criteria for testing statistical hypotheses. However, the book [1] does not answer the questions formulated above. Moreover, one should be careful while using it because of a large number of errors made in the description of the tests and examples of their application.

The assumption of normal distribution law of noise is a prerequisite for the correct application of parametric tests. The given assumption is by no means always put into practice. The use of nonparametric tests is based on the asymptotic distribution of the statistics of these tests. With the limited sample numbers of distribution statistics of parametric and nonparametric tests may differ significantly from the corresponding limit and asymptotic distributions of statistics. In the case of nonparametric tests application problems are often exacerbated by a pronounced discrete distribution of statistics.

The purpose of this paper, on the one hand, is the desire to explore the actual properties and peculiarities of application of the various tests of randomness or absence of a trend. In this regard, the work is a continuation of the research, results of which are given in [2,3]. On the other hand, the paper is devoted to the realization of the possibility of using the set of tests for randomness and the absence of a trend and ensuring the correctness of statistical conclusions by applying these test in conditions of violation of standard assumptions. The latter implies the study of the statistics distribution of the used test in the appropriate nonstandard conditions of carrying-out a statistical analysis in an interactive mode and the subsequent use of the resulting distribution of the statistic when deciding on the results of testing the hypothesis (to calculate *p-value*).

II. STATISTICAL TESTS

When checking the absence of a trend in the mathematical expectation the problem is formulated as follows. It is assumed that temporal series of values $x_1, x_2, ..., x_n$ of mutually independent random variables with mathematical expectation $\mu_1, \mu_2, ..., \mu_n$ and the same (but unknown) variances are observed. The hypothesis tested is $H_0:\ \mu_i = \mu,\ i = 1, 2, \dots, n$, that all the sample values belong to the same population with an average μ, against competing hypotheses about the presence of a trend $H_1:\ |\mu_{i+1} - \mu_i| > 0$, $i = 1, 2, \dots, n-1$.

Similarly, we formulate the problem of absence of a trend in the characteristics of the scattering.

1. *Autocorrelation test*

If the sample $x_1, x_2, ..., x_n$ is random, then the value of each of its elements should not depend on the magnitude of the preceding and succeeding members. To test this independence statistics [4] is used

$$r_{1,n} = \frac{n\sum_{i=1}^{n-1} x_i x_{i+1} - \left(\sum_{i=1}^{n} x_i\right)^2 + n x_1 x_n}{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}.$$

When the truth of the hypothesis under test statistics $r_{1,n}$ is asymptotically normally distributed with mathematical expectation and variance

$$E[r_{1,n}] = -\frac{1}{n-1}, \quad D[r_{1,n}] = \frac{n(n-3)}{(n+1)(n-1)^2}.$$

When applied to the test normalized statistics is usually used

$$r_{1,n}^* = \frac{r_{1,n} - E[r_{1,n}]}{\sqrt{D[r_{1,n}]}}. \tag{1}$$

Hypothesis of randomness (absence of trend) is rejected at large in modulus values of statistics (1). Autocorrelation test applies to parametric tests.

Normalizing transformation of the test of this statistics are Moran (2), Ljung-Box (3), Dufor-Roy statistics (4) [1]:

$$r_{1,n}^M = (n-1)^{1/2} \frac{n\, r_{1,n} + 1}{n-2}; \tag{2}$$

$$r_{1,n}^{LB} = \left[\frac{n(n+2)}{n-1}\right]^{1/2} r_{1,n}; \tag{3}$$

$$r_{1,n}^{DR} = \left[\frac{n-1}{n(n-2)}\right]^{1/2} (n\, r_{1,n} + 1). \tag{4}$$

The hypothesis under test is rejected at large and small values of the test statistics.

2. Autocorrelation test modification

In [5] a modification of the criterion, the statistics of which is the sum of estimated coefficients of the correlation of the first and second order, is considered:

$$r_{1,2} = \frac{\sum_{i=1}^{n-1}(x_i - \bar{x})(x_{i+1} - \bar{x}) + \sum_{i=1}^{n-2}(x_i - \bar{x})(x_{i+2} - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}.$$

If the H_0 is true, distribution of the statistic is asymptotically normal with expectation and the variance:

$$E[r_{1,2}] = -\frac{2n-3}{n(n-1)},$$

$$D[r_{1,2}] = \frac{2n^4 - 13n^3 + 15n^2 + 28n - 34}{n^2(n+1)(n-1)^2}.$$

Normalized statistics has the form

$$r_{1,2}^* = \frac{r_{1,2} - E[r_{1,2}]}{\sqrt{D[r_{1,2}]}}. \tag{5}$$

Hypothesis H_0 is rejected at large in modulus values of the statistics.

3. Wald-Wolfowitz test

Wald-Wolfowitz test statistic [6] is based on the serial correlation coefficient and has the form

$$R_1 = \sum_{i=1}^{n-1} x_i x_{i+1} + x_n x_1.$$

Variable R_1 is distributed is asymptotically normal with mathematical expectation

$$E[R_1] = (S_1^2 - S_2)/(n-1)$$

and variance

$$D[R_1] = \frac{S_2^2 - S_4}{n-1} + \frac{(S_1^2 - S_2)^2}{(n-1)^2} - \\ - \frac{S_1^4 - 4S_1^2 S_2 + 4S_1 S_3 + S_2^2 - 2S_4}{(n-1)(n-2)},$$

where $S_r = x_1^r + ... + x_n^r$.

Normalized statistics

$$R_1^* = \frac{R_1 - E[R_1]}{\sqrt{D[R_1]}} \tag{6}$$

is asymptotically distributed according to the standard normal law $N(0,1)$.

Wald-Wolfowitz test is a parametric test. The hypothesis H_0 is rejected at large in modulus values of the statistics.

4. Wald-Wolfowitz rank test

Let R_i be measurement rank of x_i in an ordered series of ascending values $x_1, x_2, ..., x_n$. Wald-Wolfowitz rank test statistics of serial correlation has the form [6]:

$$R = \sum_{i=1}^{n-1}\left(R_i - \frac{n+1}{2}\right)\left(R_{i+1} - \frac{n+1}{2}\right).$$

Distribution of the statistics R is asymptotically normal with parameters

$$E[R] = 0, \quad D[R] = \frac{n^2(n+1)(n-3)(5n+6)}{720}.$$

The hypothesis H_0 is rejected at large in modulus values of the statistics

$$R^* = \frac{R}{\sqrt{D[R]}}. \tag{7}$$

5. Inversion test

Inversion occurs when the sample values $x_1, x_2, ..., x_n$, are recorded in the order of their appearance after some value x_i smaller in magnitude follow, i.e. $x_i > x_j$, where $i < j \le n$. Test statistics of randomness is the total number of inversions I in sample $x_1, x_2, ..., x_n$ [7].

Hypothesis of randomness is not rejected if $I_{\alpha/2} < I < I_{1-\alpha/2}$. Possible number of inversions depends on the sample size. Mathematical expectation and variance of statistics I have the form $E[I] = n(n-1)/4$, $D[I] = (2n^3 + 3n^2 - 5n)/72$ [7].

Normalized statistics

$$I^* = \frac{I - E[I]}{\sqrt{D[I]}} \tag{8}$$

is approximately described by the standard normal law. The hypothesis H_0 is rejected at large in modulus values of the statistics $\left|I^*\right| \geq U_{1-\alpha/2}$.

Inversion test is nonparametric and distribution law of random components x_i does not affect the distribution of its statistics.

Sometimes a test with T statistics is considered, which defines the number of reversed inversions ($x_i < x_j$, $i < j$), or the test with $K = T - I$ statistics.

6. Cox-Stuart test

Cox-Stuart nonparametric test [8[8]] can be used to verify the sequence of measurements to determine the presence of a trend in the mean, as well as in the variance.

To test the hypothesis of absence of a trend in the mean values for the sample volume n the test is used with statistics

$$S_1 = \sum_{i=1}^{[n/2]} (n - 2i + 1) h_{i,n-i+1} ,$$

where $h_{i,j} = 1$, if $x_i > x_j$, and $h_{i,j} = 0$, if $x_i \leq x_j$ ($i < j$).

Normalized statistics

$$S_1^* = \frac{S_1 - E[S_1]}{\sqrt{D[S_1]}} , \tag{9}$$

where $E[S_1] = \dfrac{n^2}{8}$, $D[S_1] = \dfrac{n(n^2-1)}{24}$, when the hypothesis H_0 under test is true approximately described by the standard normal law.

7. Foster-Stuart test

This nonparametric test can be used to test hypotheses of absence of a trend in the mean values, as well as in the variances. To test the hypothesis of absence of a trend in the mean values the test is used with statistics [9] $d = \sum_{i=2}^{n} d_i$, where $d_i = u_i - l_i$; $u_i = 1$, if $x_i > x_{i-1}, x_{i-2}, ..., x_1$, whereas $u_i = 0$; $l_i = 1$, if $x_i < x_{i-1}, x_{i-2}, ..., x_1$, whereas $l_i = 0$; the range of values $-(n-1) \leq d \leq n-1$.

In the trend absence the normalized statistics

$$t = \frac{d}{\hat{\sigma}_d} , \tag{10}$$

$$\hat{\sigma}_d = \sqrt{\mu} \approx \sqrt{2\ln n - 0{,}8456} ,$$

is approximately described by Student's distribution with $\nu = n$ degrees of freedom. The hypothesis H_0 under test is rejected at large in modulus values of the statistics (10).

8. Bartels test

Suppose that in the sequence of n measurements R_i is rank i of observation x_i. Bartels [10] offered rank test of randomness of series based on statistics

$$B = \frac{\sum_{i=1}^{n-1} (R_i - R_{i+1})^2}{\sum_{i=1}^{n} (R_i - \bar{R})^2} .$$

The hypothesis H_0 under test is rejected at large in modulus values of the statistics

$$B^* = \frac{B - E[B]}{\sqrt{D[B]}} = \frac{B - 2}{2\sqrt{5/(5n+7)}} , \tag{11}$$

that in the absence of the trend approximately obeys the standard normal law.

9. Hollin test

Rank-sign test offered by Hollin is based on statistics [11]

$$r = \frac{1}{k(n-1)} \sum_{i=2}^{n} \delta\left[(x_i - \tilde{x})(x_{i-1} - \tilde{x})\right] R_i R_{i-1} , \tag{12}$$

where k is the coefficient depending on the sample size (recommended values of k are given in the table I); $\tilde{x}$ – median of set of variate values $x_{(1)} \leq x_{(2)} \leq ... \leq x_{(n)}$; R_i is rank of values $z_i = \left|x_i - \tilde{x}\right|$ in an ordered series of ascending values $z_1 \leq z_2 \leq ... \leq z_n$;

$$\delta(y) = \begin{cases} 1, & y > 0; \\ -1, & y < 0; \\ 0, & y = 0. \end{cases}$$

TABLE I

VALUES k OF HOLLIN RANK-SIGN TEST

n	5	10	20	50	100	200	400
k	10.11	36.95	140.62	851.62	3370	13407	53480

The series of values x_i is recognized as random, if $|r| < r_\alpha$. Critical values r_α can be found, for example, in [1].

10. Wald-Wolfowitz series test

A series is considered to be a sequence of elements of one or several samples in a united sample sorted by ascending limited from both sides by elements of another sample (on sequence boundary from one side).

Consider having sample value of a random variable x in order of appearance; $\tilde{x}$ is the sample median.

Values $x_i \ge \tilde{x}$ will be denoted as a, and values $x_i < \tilde{x}$ will be denoted as b. Then the test statistic [12] is N - the total number of series elements a and b.

Hypothesis of randomness series is not rejected with probability α if $n_1(\alpha) < N < n_2(\alpha)$, otherwise it is rejected in favor of an alternative non-random series. Critical values of $n_1(\alpha)$ and $n_2(\alpha)$ can be found in [13].

Normalized statistics has the form

$$N_S^* = \frac{N_s - (2n_a n_b / (n_a + n_b) + 1)}{\sqrt{2n_a n_b (2n_a n_b - n_a - n_b) / ((n_a + n_b)^2 (n_a + n_b - 1))}},$$

where n_a and n_b are respectively the number of elements of a and b of the original sequence. As an approximate distribution of the statistic the standard normal law is used. Testable hypothesis is rejected with a significance level α if $|N_s^*| > u_{1-\alpha/2}$.

11. Ramachandran-Ranganathan test

This nonparametric test takes into account not only the quantity but also the length of the series (the number of cells in series). The test statistics [1] has the form

$$RR = \sum_j j^2 n_j ,$$

where j is the length of series, n is the sample size, n_j is the number of series in length j.

The hypothesis of randomness is rejected for large values of statistics RR.

III. EXPERIMENTAL RESULTS

The distribution of the autocorrelation test statistics $r_{1,n}^*$ rapidly converges to the asymptotic law. In practice, the difference of distribution of statistics (1) from the standard normal law can be neglected for $n > 30$ (see. Fig. 1).

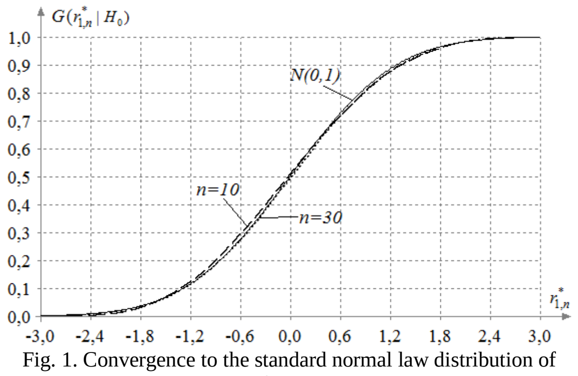

Fig. 1. Convergence to the standard normal law distribution of statistics (1) of autocorrelation test

With a strong asymmetry of the distribution law of random variables (for example, in the case of exponential law) the distribution of the statistics becomes different from the "classical" (see Fig. 2). At the same time, the asymmetry of the law affects the distribution of the statistics less significantly than the "heaviness" of tails. If the samples x_1, ..., x_n belong to asymmetric laws of extreme values (minimum or maximum), the distributions of statistics do not practically differ from the "classical".

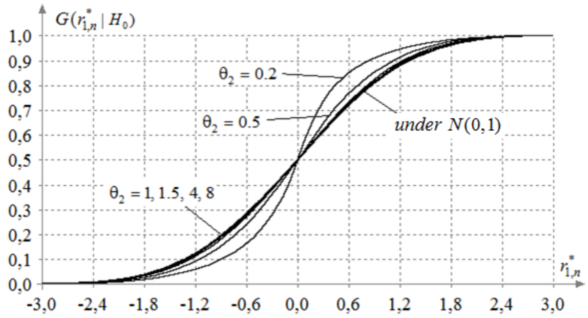

Fig. 2. The statistics distribution functions (1) depending on the shape parameter of generalized normal law (13) for $n = 25$

$$f(x) = \frac{\theta_2}{2\theta_1 \Gamma(1/\theta_2)} \exp\left(-\left(\frac{|x - \theta_0|}{\theta_1} \right)^{\theta_2} \right) \quad (13)$$

Investigation of the distribution of Ljung-Box statistics has shown that it converges to the standard normal law very slowly. Using the normal law to determine the attained significance level with limited sample sizes leads to great mistakes and, therefore, incorrect conclusions.

Distributions of Moran and Dufor-Roy statistics are in good agreement with the standard normal law for $n > 40$ and for $n > 17$ as well. When using the true distributions of statistics the result of testing the hypothesis will lead to the same attained significance level $P\{S > S^*\}$; relative to a given competing hypothesis the power of tests will be the same.

Convergence of the distribution of the modified autocorrelation test statistics (5) to the standard normal law $\mathrm{N}(0,1)$ when performing standard assumption is illustrated in Fig. 3.The studies have shown that the difference of distribution of statistics (5) from the standard normal law can be neglected only for $n > 200$. Deviations in the distribution of statistics from the standard normal law are essential with asymmetry in the observed laws and "heavy tails."

The study of distribution of Wald-Wolfowitz test statistics (6) depending on a sample size for a case of performing the assumption of normality of the analyzed samples showed the distribution agreement of statistics R_1^* with standard normal law for $n > 20$. When violating basic assumptions of normalcy of the input data, it is possible to make a conclusion about the relative stability of the distribution statistics, which is illustrated in Fig. 4. Deviations of the distribution of statistics from the standard normal law become significant when asymmetry in observed laws and "heavy tails" takes place.

Distribution of Wald-Wolfowitz rank test statistics (7) is shifted with respect to the asymptotic and is converging very slowly to the standard normal distribution law (see Fig. 5). Even for n=700 agreement with $\mathrm{N}(0,1)$ is not achieved. Discreteness of distribution of statistics can be practically negligible.

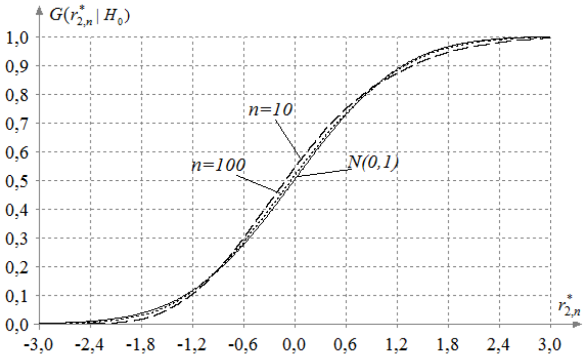

Fig. 3. Convergence to the standard normal distribution law of statistics (5)

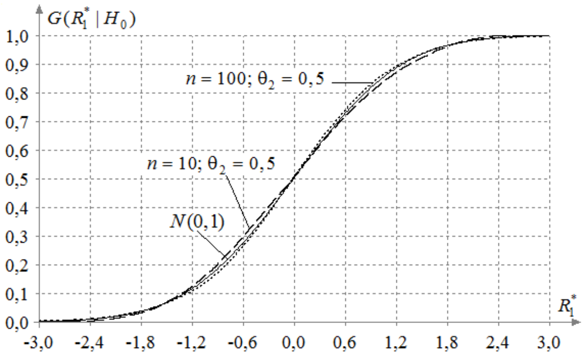

Fig. 4 The statistics distribution functions (6) depending on the shape parameter of generalized normal law (13) for $n = 25$

Using the standard normal law as the limited distribution may lead to a significant error while making decision. To prevent errors of judgment it is appropriate to use the actual distribution of statistics, which can be obtained as a consequence of the simulation for a given sample size and a specific distribution law of the observed random variables. Such simulation for all tests considered in the paper is implemented in a developed software system by "Interval Statistics for Windows».

As a result of the study of distributions of statistics R^* at various sample sizes n we proposed modification. The distribution of statistics modified

$$R^{**} = R^* + 1.1216 n^{-0.523}$$

agrees well with the standard normal law already for $n > 10$.

For $n \geq 30$ sample sizes discreteness of statistics distribution of normalized inversion test I^* can be practically negligible and rely on the standard normal distribution law as the distribution of statistics.

For small sample sizes n distribution of Cox-Stuart statistics (9) is discrete and differs greatly from the standard normal law (see Fig. 6). For $n > 40$ the difference of distribution of statistics (9) from the standard normal law can be practically neglected.

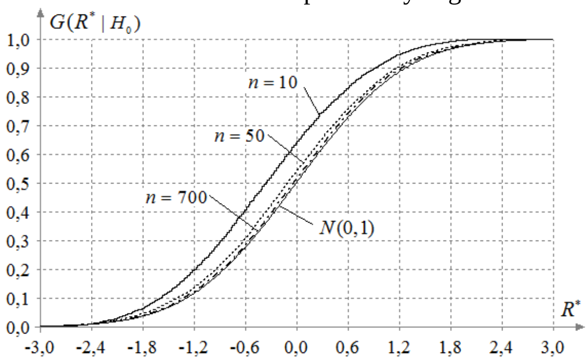

Fig. 5. Convergence to the standard normal distribution law of statistics (7)

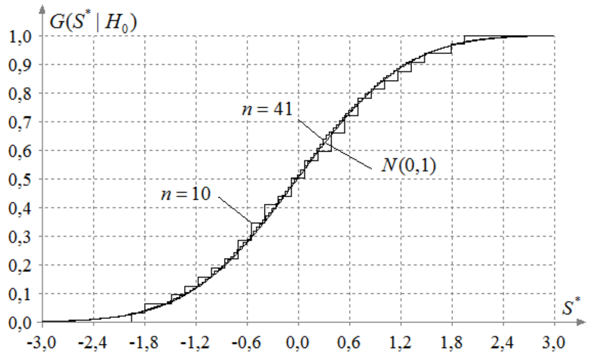

Fig. 6. Convergence to the standard normal distribution law of the function of statistics distribution (9) of Cox-Stuart test for detecting a trend in the average

If the actual sample size is less than 40, to calculate attained significance level corresponding to the obtained value of statistics S_1^*, it is advisable to use true distribution of statistics that for a given sample size n can be in a result of simulation.

As studies have shown even with rather large sample sizes ($n = 100,\ 200$) Foster-Stuart discrete distribution of statistics t differ significantly from the Student distribution with n degrees of freedom. For example, Fig. 7 illustrates the functions of statistics distribution (10).

It is shown that the distribution of the statistics (11) of nonparametric Bartels test fairly quickly converges to the standard normal distribution law, and for n >10 its difference from the standard normal law can be practically neglected.

The distribution of test statistics (12) depending on the sample size is shown in Fig. 8. The distribution of test statistics is not symmetrical with respect to 0. Therefore, when using the test it is necessary to bear in mind that due to the asymmetrical distribution of statistics the usage presented in [1] of percentage points can cause errors of deviations of correct hypothesis of randomness.

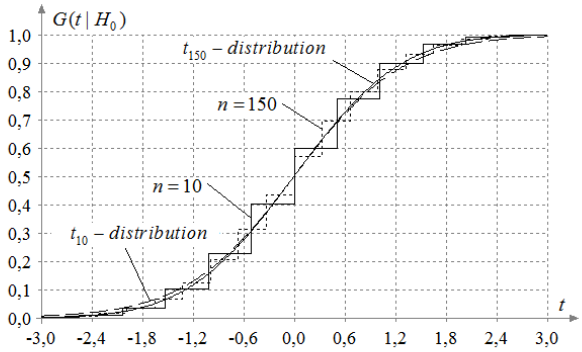

Fig. 7. The statistics distribution functions (10) at different sample sizes

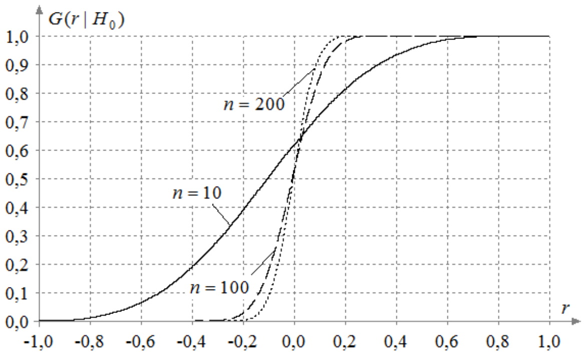

Fig. 8. The distribution functions of Hollin test statistics (12) at different sample sizes

In conditions of symmetry significant deviations of observed law from the normal practically no influence the distribution of the test statistics. However, the asymmetry of the distribution law of errors strongly affects the distribution of the test statistic (nonparametric property is lost!) (see Fig. 9).

Research of statistics distributions of Wald-Wolfowitz series test showed that even with large sample sizes ($n = 700$) the distribution of the statistic is essentially discrete and therefore is badly reconciled with the standard normal law (see Fig. 10 , Table. II).

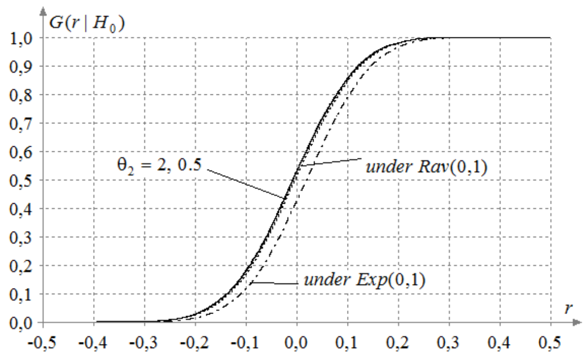

Fig. 9. The distribution functions of Hollin test statistics (12) for $n = 100$

Discrete distributions Ramachandran-Ranganathan test statistic essentially depend on the sample size.

TABLE II
TESTING CONSENT OF EMPIRICAL DISTRIBUTION OF WALD-WOLFOWITZ SERIES TEST WITH THE STANDARD NORMAL LAW

n	Goodness-of-fit test	$P\{S > S^*\}$
700	χ^2 Pearson	1.2e-08
	Kolmogorov	4.4e-05
	ω^2 Cramer-von Mises-Smirnov	0.0065
	Ω^2 Anderson-Darling	0.0055

Therefore, to calculate the achieved significance level it is appropriate to use the true distribution of the statistic. This distribution may be found, for example, by an interactive modeling with a given sample volume.

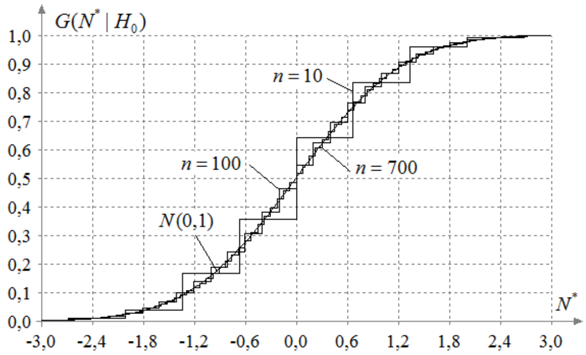

Fig. 10. . Convergence to the standard normal law distribution of statistics of Wald-Wolfowitz series test

IV. DISCUSSION OF THE RESULTS

The power of tests was compared on the small sample sizes n=10, 25, 50, 100. Empirical distribution of the test statistic corresponding to the tested and competing hypotheses, by which assessment of power were being found, obtain an acceptable accuracy were built by 1 000 000 tests.

The analysis of the power of tests was held for the situation of belonging of observed random variables to the normal law. The performance of the assumption of an independence of observed random variables corresponds to the hypothesis under test H_0 (absence of a trend). Different situations in the presence of a trend were considered as competing hypotheses.

The presence of a linear trend in the observed values modeled in accordance with the relation

$$x_i = a \cdot t + \xi_i$$

where ξ_i are independent random variables distributed according to a given law $t \in [0,1]$. The parameter value $a = 0$ corresponds to the true hypothesis under test H_0.

Values x_i were calculated in according to the equation $x_i = a \cdot (i-1)\Delta t + \xi_i$, where step Δt was defined as $\Delta t = 1/n$ depending on sample size n. Pseudorandom values ξ_i were generated in accordance with a standard normal distribution law. The power of tests with respect to competing hypotheses with the linear trend was investigated, specified by a parameter a = 0.5; 4. Corresponding competing hypotheses are indicated hereinafter as H_1, H_2. Examples of time series in trend with parameter $a = 0.5$ and $a = 4$ with sample size $n = 100$ are shown in Fig. 11.

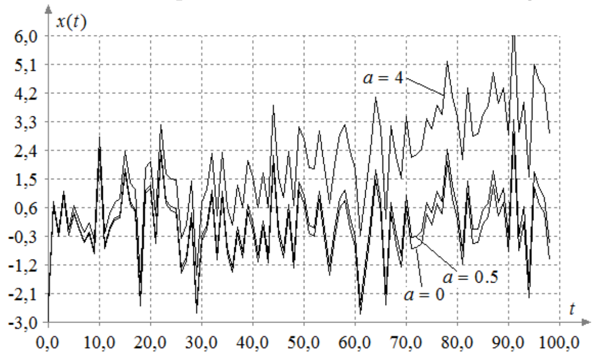

Fig. 11. Linear trend with $a = 0, 0.5, 4$

Received assessment of power of tests with respect to competing hypotheses H_1, H_2, for a number of sample sizes are shown in Table. III - VIII.

With a linear trend autocorrelation test significantly loses in power than its modification which is the sum of the coefficients of correlation of the first and second order.

TABLE III
THE POWER OF AUTOCORRELATION TEST AND ITS MODIFICATION

n	α	Autocorrelation test		Modified autocorrelation test	
		H_1	H_2	H_1	H_2
10	0.1	0.103	0.295	0.109	0.708
	0.05	0.051	0.175	0.056	0.570
	0.025	0.026	0.097	0.029	0.435
	0.01	0.011	0.043	0.012	0.273
25	0.1	0.105	0.837	0.113	0.980
	0.05	0.053	0.739	0.059	0.958
	0.025	0.027	0.631	0.031	0.921
	0.01	0.011	0.483	0.013	0.851
50	0.1	0.108	0.993	0.117	1.000
	0.05	0.056	0.985	0.062	1.000
	0.025	0.029	0.969	0.033	0.999
	0.01	0.012	0.935	0.014	0.997
100	0.1	0.113	1.000	0.125	1.000
	0.05	0.058	1.000	0.067	1.000
	0.025	0.030	1.000	0.036	1.000
	0.01	0.013	1.000	0.016	1.000

Power of Wald-Wolfowitz test also somewhat lower power than its nonparametric analogue (see Table. IV).

Inversion tests with statistics I, I^* slightly inferior according to the power than inversion tests with statistics K, T (see Table V). The poser of test is rather high even with a small sample size and tends rapidly to unity.

Note that the distribution of Foster-Stuart, Stuart-Cox test statistic are discrete that complicates assessment of power and analysis of results. Therefore, during research of distribution of these tests were approximated by the normal law, i.e. were obtained "asymptotic power".

TABLE IV
THE POWER OF WALD-WOLFOWITZ TEST AND WALD-WOLFOWITZ RANK TEST

n	α	Wald-Wolfowitz test		Wald-Wolfowitz rank test	
		H_1	H_2	H_1	H_2
10	0.1	0.102	0.276	0.105	0.547
	0.05	0.051	0.156	0.052	0.400
	0.025	0.026	0.083	0.028	0.276
	0.01	0.010	0.035	0.011	0.143
25	0.1	0.105	0.835	0.107	0.921
	0.05	0.053	0.735	0.055	0.862
	0.025	0.027	0.624	0.028	0.787
	0.01	0.011	0.475	0.012	0.673
50	0.1	0.108	0.993	0.109	0.997
	0.05	0.056	0.984	0.056	0.993
	0.025	0.029	0.969	0.029	0.986
	0.01	0.012	0.935	0.012	0.968
100	0.1	0.113	1.000	0.112	1.000
	0.05	0.058	1.000	0.059	1.000
	0.025	0.030	1.000	0.030	1.000
	0.01	0.013	1.000	0.013	1.000

TABLE V
THE POWER OF INVERSION TEST

n	α	Inversion test I, I^*		Inversion test K, T	
		H_1	H_2	H_1	H_2
10	0.1	0.102	0.883	0.128	0.926
	0.05	0.068	0.822	0.089	0.883
	0.025	0.026	0.637	0.037	0.740
	0.01	0.015	0.517	0.022	0.637
25	0.1	0.173	1.000	0.185	1.000
	0.05	0.097	0.999	0.104	1.000
	0.025	0.054	0.998	0.059	0.998
	0.01	0.027	0.994	0.031	0.995
50	0.1	0.253	1.000	0.258	1.000
	0.05	0.163	1.000	0.167	1.000
	0.025	0.100	1.000	0.103	1.000
	0.01	0.053	1.000	0.055	1.000
100	0.1	0.401	1.000	0.403	1.000
	0.05	0.283	1.000	0.285	1.000
	0.025	0.195	1.000	0.196	1.000
	0.01	0.116	1.000	0.117	1.000

TABLE VI
THE POWER OF FOSTER-STUART, COX-STUART TESTS

n	α	Stuart-Cox test		Foster-Stuart test	
		H_1	H_2	H_1	H_2
10	0.1	0.114	0.515	0.100	0.491
	0.05	0.057	0.190	0.050	0.301
	0.025	0.028	0.041	0.025	0.176
	0.01	0.012	0.003	0.010	0.063
25	0.1	0.154	0.984	0.104	0.679
	0.05	0.084	0.953	0.050	0.530
	0.025	0.049	0.906	0.025	0.418
	0.01	0.020	0.812	0.010	0.270
50	0.1	0.206	1.000	0.112	0.756
	0.05	0.123	1.000	0.055	0.636
	0.025	0.075	0.999	0.029	0.537
	0.01	0.039	0.997	0.011	0.401
100	0.1	0.308	1.000	0.117	0.798
	0.05	0.210	1.000	0.059	0.699
	0.025	0.135	1.000	0.033	0.610
	0.01	0.077	1.000	0.012	0.480

TABLE VII
THE POWER OF BARRTELS TEST

n	α	Bartels test		Wald-Wolfowitz series test	
		H_1	H_2	H_1	H_2
10	0.1	0.105	0.626	0.103	0.348
	0.05	0.053	0.493	0.052	0.237
	0.025	0.028	0.381	0.026	0.152
	0.01	0.012	0.250	0.010	0.090
25	0.1	0.107	0.937	0.110	0.687
	0.05	0.055	0.890	0.056	0.565
	0.025	0.028	0.827	0.027	0.446
	0.01	0.012	0.728	0.012	0.320
50	0.1	0.109	0.998	0.106	0.927
	0.05	0.056	0.995	0.053	0.873
	0.025	0.029	0.989	0.026	0.796
	0.01	0.012	0.975	0.010	0.687
100	0.1	0.112	1.000	0.109	0.997
	0.05	0.059	1.000	0.055	0.994
	0.025	0.031	1.000	0.027	0.986
	0.01	0.013	1.000	0.010	0.965

TABLE VIII
THE POWER OF HOLLIN, RAMACHANDRAN-RANGANATHAN TESTS

n	α	Hollin test		Ramachandran-Ranganathan test	
		H_1	H_2	H_1	H_2
10	0.1	0.105	0.540	0.109	0.510
	0.05	0.053	0.414	0.056	0.422
	0.025	0.027	0.289	0.029	0.359
	0.01	0.011	0.148	0.011	0.282
25	0.1	0.111	0.887	0.130	0.846
	0.05	0.059	0.806	0.070	0.808
	0.025	0.029	0.714	0.039	0.779
	0.01	0.011	0.593	0.017	0.737
50	0.1	0.113	0.997	0.133	0.948
	0.05	0.057	0.993	0.072	0.936
	0.025	0.028	0.984	0.042	0.928
	0.01	0.011	0.972	0.019	0.914
100	0.1	0.116	1.000	0.139	0.984
	0.05	0.058	1.000	0.076	0.980
	0.025	0.030	1.000	0.047	0.977
	0.01	0.012	1.000	0.020	0.974

Summarizing the results for the detection of a linear trend in the average the following tests can be recommended (tests are ranked in order of preference):

Inversions $(K, T, I^*, I) \succ$ Cox-Stuart (S_1^*) $\succ$ modification of the autocorrelation test $(r_{1,2}^*) \succ$ Bartels $(B^*) \succ$ Wald-Wolfowitz rank (R^*), Hollin $(r) \succ$ Ramachandran-Ranganathan $(RR) \succ$ Wald-Wolfowitz (R_1^*), autocorrelation $(r_{1,n}^*) \succ$ Wald-Wolfowitz series test $(N^2) \succ$ Foster-Stuart (t).

V. SUMMARY AND CONCLUSIONS

To ensure the correctness of statistical conclusions using tests considered in situations when the standard assumptions, are violated providing the legitimacy of using the classical results, or no information about the "true" distribution of the statistic of a used tests (under specific conditions and at a particular sample size), an interactive mode of research of statistic distributions with the following usage of the resulting distribution when deciding on the results of testing the hypothesis (for calculation p-value) is released.

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Veretel`nikova Irina V.,
Undergraduate, Novosibirsk State Technical University, Department of Applied Mathematics

Lemeshko Boris Yu.,
Professor, Novosibirsk State Technical University, Department of Applied Mathematics