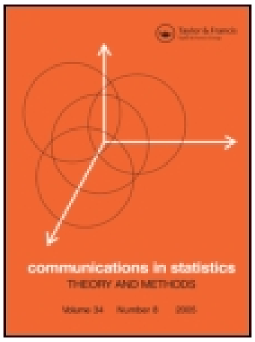

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Models of Statistic Distributions of Nonparametric Goodness-of-Fit Tests in Composite Hypotheses Testing for Double Exponential Law Cases

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In this article, are presented more accurate results (tables of percentage points and statistic distribution models) for the nonparametric goodness-of-fit tests in testing composite hypotheses using the maximum likelihood estimate (MLE) for the double exponential distribution law. The statistic distributions of the nonparametric goodness-of-fit tests are investigated by applying the statistical simulation methods.

Keywords Anderson-Darling test; Composite hypotheses testing; Cramer-Von Mises-Smirnov test; Double exponential distribution; Goodness-of-fit test; Kolmogorov test.

Mathematics Subject Classification 62G10.

1. Introduction

In composite hypotheses testing of the form H_0: $F(x) \in \{F(x, \theta), \theta \in \Theta\}$, when the estimate $\hat{\theta}$ of the scalar or vector distribution parameter $F(x, \theta)$ is calculated using the same sample, the Kolmogorov, ω^2 Cramer-Von Mises-Smirnov, and Ω^2 Anderson-Darling the nonparametric goodness-of-fit tests lose the free distribution property.

The value

$$D_n = \sup_{|x|<\infty} |F_n(x) - F(x, \theta)|,$$

where $F_n(x)$ is a empirical distribution function, n is a sample size, is used in the Kolmogorov test as a distance between the empirical and theoretical laws. In testing hypotheses, a statistic with Bolshev correction (Bolshev, 1987) of the form

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(Bolshev and Smirnov, 1983)

$$S_K = \frac{6nD_n + 1}{6\sqrt{n}}, \tag{1}$$

where $D_n = \max(D_n^+, D_n^-)$,

$$D_n^+ = \max_{1\le i\le n}\left\{\frac{i}{n} - F(x_i, \theta)\right\}, \quad D_n^- = \max_{1\le i\le n}\left\{F(x_i, \theta) - \frac{i-1}{n}\right\},$$

n is a sample size, $x_1, x_2, \ldots, x_n$ are sample values in increasing order is usually used. The limiting probability distribution function (pdf) of the statistic (1) in testing simple hypotheses follows the Kolmogorov pdf $K(S) = \sum_{k=-\infty}^{\infty}(-1)^k e^{-2k^2s^2}$.

In the ω^2 Cramer-Von Mises-Smirnov test, one uses the statistic of the form

$$S_\omega = n\omega_n^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left\{F(x_i, \theta) - \frac{2i-1}{2n}\right\}^2, \tag{2}$$

and in the test of the Ω^2 Anderson-Darling type (Anderson and Darling, 1952, 1954), the statistic of the form

$$S_\Omega = -n - 2\sum_{i=1}^{n}\left\{\frac{2i-1}{2n}\ln F(x_i, \theta) + \left(1 - \frac{2i-1}{2n}\right)\ln(1 - F(x_i, \theta))\right\} \tag{3}$$

is used. In testing a simple hypothesis, the statistic (2) follows the pdf (Bolshev and Smirnov, 1983) of the form

$$a1(S) = \frac{1}{\sqrt{2s}}\sum_{j=0}^{\infty}\frac{\Gamma(j+1/2)\sqrt{4j+1}}{\Gamma(1/2)\Gamma(j+1)}\exp\left\{-\frac{(4j+1)^2}{16S}\right\}$$
$$\times\left\{I_{-\frac{1}{4}}\left[\frac{(4j+1)^2}{16S}\right] - I_{\frac{1}{4}}\left[\frac{(4j+1)^2}{16S}\right]\right\}$$

where $I_{-\frac{1}{4}}(\cdot)$, $I_{\frac{1}{4}}(\cdot)$ is a modified Bessel function,

$$I_v(z) = \sum_{k=0}^{\infty}\frac{\left(\frac{z}{2}\right)^{v+2k}}{\Gamma(k+1)\Gamma(k+v+1)}, \quad |z| < \infty, \quad |\arg z| < \pi,$$

and the statistic (3) follows the pdf (Bolshev and Smirnov, 1983) of the form

$$a2(S) = \frac{\sqrt{2\pi}}{S}\sum_{j=0}^{\infty}(-1)^j\frac{\Gamma\left(j+\frac{1}{2}\right)(4j+1)}{\Gamma\left(\frac{1}{2}\right)\Gamma(j+1)}\exp\left\{-\frac{(4j+1)^2\pi^2}{8S}\right\}$$
$$\times\int_0^{\infty}\exp\left\{\frac{S}{8(y^2+1)} - \frac{(4j+1)^2\pi^2y^2}{8S}\right\}dy.$$

In composite hypotheses testing, the conditional distribution law of the statistic $G(S \mid H_0)$ is affected by a number of factors: the form of the observed law $F(x, \theta)$

corresponding to the true hypothesis H_0; the type of the parameter estimated and the number of parameters to be estimated; and in some cases, the specific value of the parameter (e.g., in the case of gamma-distribution and beta-distribution families); the method of parameter estimation. The distinctions in the limiting distributions of the same statistics in testing simple and composite hypotheses are too significant to be neglected.

Kac et al. (1955) was a pioneer in investigating the statistic distributions of the nonparametric goodness-of-fit tests in testing composite hypotheses. Then, to solve this problem, various approaches were used see (Darling, 1955, 1957; Durbin, 1973, 1975, 1976; Gihman, 1953, 1961; Martynov, 1978; Pearson and Hartley, 1972; Stephens, 1970, 1974; Chandra et al., 1981; Tyurin, 1984; Tyurin and Savvushkina, 1984; Dzhaparidze and Nikulin, 1982; Nikulin, 1992a,b).

In our research (Lemeshko and Postovalov, 1998, 2001a,b, 2002; Lemeshko and Maklakov, 2004; Lemeshko, 2004; Lemeshko et al., 2010b) the statistic distributions of the nonparametric goodness-of-fit tests are investigated using the methods of statistical simulation, and the approximate law models for constructed empirical distributions are found. The results obtained were used to develop recommendations for standardization (R 50.1.037-2002, 2002).

2. Statistic Distributions of Tests in Testing Composite Hypotheses Concerning Double Exponential Law

In testing composite hypotheses for the distribution law with the density

$$f(x, \boldsymbol{\theta}) = \frac{\theta_0}{2\theta_1\Gamma(1/\theta_0)} \exp\left\{-\left(\frac{|x-\theta_2|}{\theta_1}\right)^{\theta_0}\right\}, \tag{4}$$

the distributions $G(S \mid H_0)$ of nonparametric goodness-of-fit test statistics depend not only on the fact of estimating the parameters involved (θ_0, θ_1, or θ_2), but also on the specific value of the shape parameter θ_0 (Lemeshko and Maklakov, 2004).

The family (4) defines a set of symmetric laws, the special cases of which are the normal distribution ($\theta_0 = 2$), and the Laplace distribution ($\theta_0 = 1$). Sometimes this distribution is called double-sided exponential law, although usually $\theta_0 = 1$ is implied.

The feature in the behavior of the nonparametric goodness-of-fit tests statistics $G(S \mid H_0)S_\omega$ when testing composite hypotheses for the family (4) is that with the shape parameter increasing up to $\theta_0 \approx 1.64$, the distributions $G(S \mid H_0)$ are shifting to the right, and when this value is exceeded the shift starts in the opposite direction (see Fig. 1).

The upper percentage points obtained as a result of statistic simulation, and the constructed models of the limiting statistic distributions of the Kolmogorov, Cramer-Von Mises-Smirnov, and Anderson-Darling tests for the values of the shape parameter $\theta_0 = 0.5$, 0.75, 1, 1.6, 2, 3, 4, 5, 7 when using MLEs are presented in Tables 1–9. These results refine and expand the results presented in Lemeshko and Maklakov (2004). If the value of the shape parameter θ_0 is not congruent with the values given in the tables, interpolation could be used to obtain percentage point approximations.

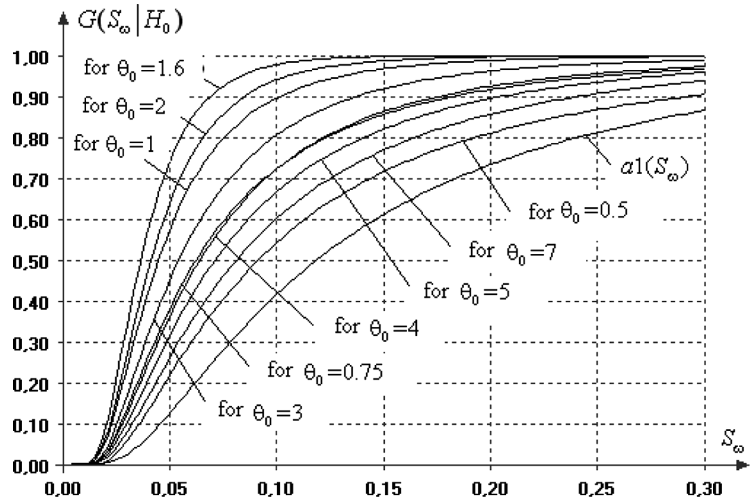

Figure 1. Statistic distributions (2) of Cramer-Von Mises-Smirnov goodness-of-fit tests in testing composite hypotheses concerning family (4) if MLE is used for all three parameters depending on the value θ_0.

Table 1
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 0.5$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.184	1.322	1.596	γ(3.7437, 0.1349, 0.325)
θ_1	1.165	1.303	1.578	γ(3.5811, 0.1366, 0.325)
θ_2	1.182	1.308	1.560	γ(4.4361, 0.1186, 0.320)
θ_0, θ_1	1.123	1.259	1.534	γ(3.1115, 0.1442, 0.330)
θ_0, θ_2	1.144	1.271	1.528	γ(3.8417, 0.1265, 0.322)
θ_1, θ_2	1.110	1.233	1.480	γ(3.6713, 0.1251, 0.326)
θ_0, θ_1, θ_2	1.129	1.255	1.508	B_3(4.4961, 5.7241, 3.1229, 2.26825, 0.306)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.325	0.441	0.723	B_3(2.6596, 1.5374, 22.6346, 1.100, 0.015)
θ_1	0.321	0.435	0.718	B_3(2.3196, 1.5425, 22.7256, 1.2000, 0.016)
θ_2	0.318	0.421	0.676	B_3(2.8412, 1.9552, 17.4052, 1.200, 0.014)
θ_0, θ_1	0.313	0.428	0.711	B_3(1.6693, 1.3771, 15.5765, 0.940, 0.017)
θ_0, θ_2	0.300	0.405	0.664	B_3(2.4600, 1.7966, 19.8161, 1.20, 0.014)
θ_1, θ_2	0.286	0.388	0.637	B_3(3.8085, 1.5324, 32.1564, 0.950, 0.011)
θ_0, θ_1, θ_2	0.295	0.399	0.656	B_3(3.0778, 1.6214, 30.1798, 1.2, 0.013)

(*continued*)

Table 1
Continued

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Anderson-Darling test		
θ_0	1.735	2.303	3.697	$B_3(5.1673, 1.7964, 33.1733, 6.000, 0.088)$
θ_1	1.718	2.286	3.676	$B_3(5.3595, 1.7388, 37.1241, 6.000, 0.087)$
θ_2	1.819	2.335	3.617	$B_3(3.4953, 2.2898, 14.9125, 6.400, 0.116)$
θ_0, θ_1	1.671	2.238	3.633	$B_3(5.786, 1.500, 45.3895, 5.200, 0.08)$
θ_0, θ_2	1.631	2.159	3.454	$B_3(3.1191, 2.0392, 20.4775, 6.600, 0.116)$
θ_1, θ_2	1.578	2.093	3.356	$B_3(3.0953, 2.0351, 22.1953, 6.800, 0.118)$
$\theta_0, \theta_1, \theta_2$	1.608	2.132	3.416	$B_3(4.5039, 2.0396, 37.0448, 8.000, 0.092)$

Table 2
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 0.75$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.196	1.333	1.605	$\gamma(3.7808, 0.1349, 0.330)$
θ_1	1.172	1.309	1.584	$B_3(4.5525, 4.9086, 3.8651, 2.3718, 0.315)$
θ_2	1.068	1.173	1.384	$B_3(4.7066, 10.8120, 1.8954, 2.50, 0.302)$
θ_0, θ_1	1.126	1.263	1.560	$B_3(4.0450, 4.9340, 3.7586, 2.3832, 0.310)$
θ_0, θ_2	1.021	1.123	1.328	$B_3(4.9912, 6.4499, 2.6816, 1.90, 0.295)$
θ_1, θ_2	0.985	1.084	1.283	$B_3(5.5451, 7.3578, 3.0559, 2.100, 0.280)$
$\theta_0, \theta_1, \theta_2$	0.937	1.032	1.223	$B_3(4.5753, 6.8907, 2.74626, 1.8903, 0.294)$
		for Cramer-Von Mises-Smirnov test		
θ_0	0.329	0.4440	0.726	$B_3(4.9844, 1.4891, 37.5211, 1.001, 0.0085)$
θ_1	0.322	0.437	0.719	$B_3(6.1042, 1.2892, 53.3676, 0.8800, 0.009)$
θ_2	0.226	0.289	0.443	$B_3(3.5628, 2.6431, 16.5587, 1.030, 0.010)$
θ_0, θ_1	0.313	0.428	0.711	$B_3(1.6779, 1.3775, 15.6587, 0.940, 0.017)$
θ_0, θ_2	0.202	0.265	0.420	$B_3(2.5230, 2.8292, 19.5602, 1.4650, 0.014)$
θ_1, θ_2	0.192	0.255	0.408	$B_3(2.6652, 2.4143, 24.7681, 1.300, 0.013)$
$\theta_0, \theta_1, \theta_2$	0.184	0.248	0.404	$B_3(3.2636, 1.7846, 29.6713, 0.800, 0.0118)$
		for Anderson-Darling test		
θ_0	1.755	2.322	3.715	$B_3(5.5017, 1.7097, 32.6151, 5.4000, 0.09)$
θ_1	1.721	2.290	3.681	$B_3(5.7288, 1.7042, 38.1627, 5.700, 0.085)$
θ_2	1.422	1.779	2.626	$B_3(3.1406, 2.9653, 10.3579, 5.500, 0.12)$
θ_0, θ_1	1.669	2.236	3.632	$B_3(5.7330, 1.5217, 44.0784, 5.200, 0.078)$
θ_0, θ_2	1.208	1.553	2.410	$B_3(5.9765, 2.6769, 32.3123, 6.400, 0.070)$
θ_1, θ_2	1.166	1.509	2.362	$B_3(6.5437, 2.5007, 38.5262, 6.000, 0.07)$
$\theta_0, \theta_1, \theta_2$	1.116	1.465	2.322	$B_3(6.2120, 2.1027, 40.3780, 4.800, 0.075)$

Table 3
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 1$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.204	1.340	1.613	γ(3.9433, 0.1340, 0.3200)
θ_1	1.177	1.313	1.587	B_3(4.4680, 4.8450, 3.9105, 2.3784, 0.324)
θ_2	0.957	1.045	1.223	B_3(5.3541, 7.2519, 2.5630, 1.7652, 0.302)
θ_0, θ_1	1.1300	1.268	1.545	B_3(3.9724, 4.8877, 3.7872, 2.3973, 0.3150)
θ_0, θ_2	0.911	0.995	1.162	B_3(4.9365, 8.1400, 2.2383, 1.7312, 0.3)
θ_1, θ_2	0.863	0.940	1.095	γ(6.2949, 0.0624, 0.2613)
θ_0, θ_1, θ_2	0.798	0.870	1.014	γ(5.5391, 0.0606, 0.2700)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.333	0.447	0.7295	B_3(2.8981, 1.5614, 20.0694, 1.00, 0.014)
θ_1	0.323	0.438	0.719	B_3(3.9800, 1.4667, 38.0035, 1.13, 0.0111)
θ_2	0.152	0.187	0.268	B_3(3.3130, 3.8338, 10.0967, 0.7517, 0.011)
θ_0, θ_1	0.313	0.428	0.711	B_3(3.7712, 1.1413, 38.6694, 0.790, 0.011)
θ_0, θ_2	0.131	0.162	0.234	B_3(3.9062, 3.9000, 13.5396, 0.7491, 0.009)
θ_1, θ_2	0.115	0.144	0.213	B_3(4.4891, 3.7706, 17.5774, 0.7065, 0.0085)
θ_0, θ_1, θ_2	0.103	0.132	0.207	B_3(5.2856, 3.0510, 34.1638, 0.7312, 0.0079)
		for Anderson-Darling test		
θ_0	1.775	2.342	3.734	B_3(2.9208, 2.5613, 25.6028, 12.5850, 0.117)
θ_1	1.725	2.290	3.685	B_3(4.0842, 1.7532, 28.1434, 6.00, 0.105)
θ_2	1.071	1.302	1.837	B_3(4.2270, 3.0430, 8.4289, 3.000, 0.09)
θ_0, θ_1	1.668	2.235	3.630	B_3(3.7352, 1.5349, 29.4582, 5.300, 0.098)
θ_0, θ_2	0.871	1.062	1.522	B_3(4.8431, 4.1424, 14.2651, 4.6769, 0.073)
θ_1, θ_2	0.798	0.982	1.439	B_3(5.3576, 3.8690, 17.2148, 4.2386, 0.073)
θ_0, θ_1, θ_2	0.726	1.116	1.394	B_3(5.2973, 3.3781, 27.5085, 4.8145, 0.073)

Table 4
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 1.6$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.216	1.351	1.621	B_3(4.2366, 5.7254, 2.8969, 2.4200, 0.330)
θ_1	1.185	1.322	1.596	B_3(4.3698, 5.2853, 3.3545, 2.3863, 0.318)
θ_2	0.851	0.923	1.069	B_3(5.4129, 7.6381, 2.1289, 1.3936, 0.290)
θ_0, θ_1	1.141	1.280	1.557	B_3(4.9730, 4.5743, 4.6422, 2.3576, 0.29)
θ_0, θ_2	0.828	0.898	1.039	B_3(6.2506, 7.4916, 2.5914, 1.4130, 0.275)

(*continued*)

Table 4
Continued

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
θ_1, θ_2	0.770	0.831	0.953	B_3(5.3623, 7.3149, 2.1379, 1.1702, 0.29))
$\theta_0, \theta_1, \theta_2$	0.704	0.759	0.873	B_3(7.4853, 7.2752, 3.2095, 1.14609, 0.260)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.339	0.453	0.735	*Sb*(3.6139, 1.0337, 3.400, 0.013)
θ_1	0.325	0.440	0.723	*Sb*(2.7348, 0.9148, 1.800, 0.016)
θ_2	0.121	0.149	0.219	B_3(4.5239, 3.7332, 15.6889, 0.6596, 0.009)
θ_0, θ_1	0.314	0.429	0.711	*Sb*(2.3111, 0.8115, 1.350, 0.016)
θ_0, θ_2	0.109	0.134	0.194	B_3(4.2190, 3.9949, 12.6139, 0.5642, 0.0087)
θ_1, θ_2	0.087	0.104	0.143	B_3(4.5491, 4.8658, 9.0448, 0.4000, 0.008)
$\theta_0, \theta_1, \theta_2$	0.069	0.083	0.118	B_3(6.8750, 4.6392, 18.020, 0.3937, 0.006)
		for Anderson-Darling test		
θ_0	1.819	2.383	3.774	B_3(3.7982, 2.4042, 26.2612, 10.00, 0.095)
θ_1	1.735	2.304	3.697	B_3(3.6908, 2.1990, 32.1310, 10.00, 0.10)
θ_2	0.864	1.052	1.513	B_3(4.0782, 5.1594, 17.0570, 7.900, 0.09)
θ_0, θ_1	1.669	2.235	3.630	B_3(4.6625, 1.4267, 33.5120, 4.500, 0.09)
θ_0, θ_2	0.716	0.863	1.207	B_3(4.5576, 4.2326, 10.9573, 3.23142, 0.08)
θ_1, θ_2	0.589	0.695	0.941	B_3(4.5825, 5.3012, 7.9243, 2.5555, 0.0775)
$\theta_0, \theta_1, \theta_2$	0.492	0.587	0.819	B_3(5.0884, 5.2459, 10.6760, 2.4738, 0.068)

Table 5
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 2$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.219	1.354	1.624	B_3(4.6934, 5.6544, 3.0971, 2.4099, 0.315)
θ_1	1.190	1.327	1.600	B_3(4.8849, 5.2341, 3.6279, 2.3872, 0.303)
θ_2	0.888	0.963	1.114	B_3(5.2604, 7.4327, 2.1872, 1.4774, 0.30)
θ_0, θ_1	1.148	1.287	1.564	B_3(4.6127, 4.8440, 4.1337, 2.4080, 0.295)
θ_0, θ_2	0.880	0.956	1.108	B_3(5.7052, 7.2179, 2.5877, 1.5433, 0.29)
θ_1, θ_2	0.835	0.909	1.057	γ(6.4721, 0.0580, 0.2620)
$\theta_0, \theta_1, \theta_2$	0.784	0.861	1.021	B_3(9.3597, 5.7532, 5.8275, 1.4507, 0.2500)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.341	0.456	0.737	*Sb*(2.7740, 0.9495, 1.9000, 0.0170)
θ_1	0.327	0.442	0.725	*Sb*(3.3182, 0.94801, 2.9500, 0.016)
θ_2	0.134	0.165	0.238	B_3(4.4331, 3.6365, 13.9198, 0.6632, 0.0084)
θ_0, θ_1	0.315	0.430	0.712	*Sb*(2.2458, 0.7970, 1.300, 0.017)
θ_0, θ_2	0.127	0.156	0.225	B_3(4.0430, 3.72568, 12.5794, 0.6313, 0.0087)

(*continued*)

Table 5
Continued

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
θ_1, θ_2	0.103	0.126	0.178	B_3(4.1153, 4.1748, 11.0347, 0.5116, 0.009)
$\theta_0, \theta_1, \theta_2$	0.086	0.107	0.161	B_3(6.7594, 3.8575, 28.6668, 0.5921, 0.006)
		for Anderson-Darling test		
θ_0	1.842	2.404	3.796	B_3(3.0026, 2.7848, 21.7432, 12.5565, 0.111)
θ_1	1.743	2.309	3.704	B_3(3.4638, 2.3300, 35.7115, 12.6033, 0.105)
θ_2	0.892	1.087	1.552	B_3(4.1081, 5.0598, 16.9721, 7.9065, 0.09)
θ_0, θ_1	1.672	2.237	3.632	B_3(4.2125, 1.5874, 32.6127, 5.500, 0.09)
θ_0, θ_2	0.779	0.945	1.335	B_3(4.6827, 3.7977, 12.6413, 3.4486, 0.08)
θ_1, θ_2	0.630	0.750	1.032	B_3(4.7262, 4.6575, 9.4958, 2.7171, 0.0775)
$\theta_0, \theta_1, \theta_2$	0.529	0.640	0.919	B_3(4.3857, 5.7110, 17.3440, 5.0052, 0.075)

Table 6
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 3$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.222	1.357	1.626	B_3(6.5249, 5.0755, 4.5306, 2.4069, 0.285)
θ_1	1.197	1.334	1.606	B_3(5.2350, 5.0903, 3.9316, 2.3905, 0.300)
θ_2	0.998	1.095	1.291	B_3(8.5402, 6.1019, 4.4047, 1.8871, 0.250)
θ_0, θ_1	1.161	1.301	1.577	B_3(5.5689, 4.6553, 4.9339, 2.4147, 0.280)
θ_0, θ_2	0.999	1.096	1.293	B_3(6.5008, 7.8186, 3.25827, 2.1735, 0.270)
θ_1, θ_2	0.970	1.069	1.269	B_3(6.8503, 6.2212, 3.9819, 1.9216, 0.265)
$\theta_0, \theta_1, \theta_2$	0.936	1.039	1.247	γ(3.6025, 0.10128, 0.3125)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.345	0.459	0.741	B_3(3.2178, 1.6133, 19.2436, 1.000, 0.0125)
θ_1	0.330	0.445	0.727	B_3(3.6534, 1.5249, 28.5258, 1.0550, 0.0117)
θ_2	0.179	0.224	0.329	B_3(3.6203, 2.6395, 11.3638, 0.600, 0.010)
θ_0, θ_1	0.317	0.432	0.715	B_3(6.6688, 1.2016 ,63.6672, 0.830, 0.008)
θ_0, θ_2	0.177	0.222	0.329	B_3(3.5065, 2.5837, 11.5972, 0.600, 0.010)
θ_1, θ_2	0.154	0.196	0.299	B_3(3.7581, 2.3887, 13.3525, 0.500, 0.010)
$\theta_0, \theta_1, \theta_2$	0.138	0.181	0.289	*Sl*(1.1736, 1.2083, 0.1163, 0.0103)
		for Anderson-Darling test		
θ_0	1.881	2.441	3.835	B_3(3.7511, 2.3357,19.6979, 8.0000, 0.095)
θ_1	1.757	2.324	3.718	B_3(4.1218, 2.1349, 30.0763, 8.500, 0.094)
θ_2	1.049	1.282	1.823	B_3(4.6108, 3.3193, 12.0931, 4.0000, 0.079)
θ_0, θ_1	1.679	2.245	3.638	B_3(6.0616, 1.6126, 47.4733, 5.800, 0.074)
θ_0, θ_2	0.989	1.215	1.741	B_3(4.7371, 3.2610, 13.7406, 4.0000, 0.070)
θ_1, θ_2	0.819	1.009	1.472	B_3(5.2098, 3.5915, 16.7524, 4.0000, 0.070)
$\theta_0, \theta_1, \theta_2$	0.716	0.908	1.391	B_3(5.9548, 2.9777, 28.5342, 3.800, 0.069)

Table 7

Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 4$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.223	1.358	1.626	B_3(3.6243, 5.3291, 2.4503, 2.1853, 0.36)
θ_1	1.202	1.338	1.610	B_3(4.4775, 5.7536, 2.9612, 2.4028, 0.31)
θ_2	1.060	1.169	1.388	B_3(3.8031, 7.8639, 1.9955, 2.1337, 0.34)
θ_0, θ_1	1.172	1.311	1.586	B_3(2.6607, 6.1554, 2.0175, 2.4197, 0.364)
θ_0, θ_2	1.061	1.170	1.389	B_3(4.1178, 7.0193, 2.3554, 2.1116, 0.330)
θ_1, θ_2	1.039	1.150	1.372	B_3(4.4530, 6.5204, 2.8504, 2.1247, 0.315)
θ_0, θ_1, θ_2	1.013	1.126	1.353	γ(3.5001, 0.1150, 0.3200)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.346	0.460	0.742	B_3(2.6493, 2.3780, 23.7392, 2.3027, 0.0133)
θ_1	0.332	0.447	0.729	B_3(2.9074, 1.7706, 24.9344, 1.40, 0.0134)
θ_2	0.212	0.270	0.409	B_3(3.2370, 2.7787, 15.5238, 1.05, 0.011)
θ_0, θ_1	0.319	0.434	0.717	B_3(2.8323, 1.4558, 24.26690, 1.0, 0.012)
θ_0, θ_2	0.212	0.271	0.412	B_3(2.9892, 2.7082, 14.1961, 1.0, 0.0117)
θ_1, θ_2	0.193	0.250	0.390	B_3(3.7333, 2.7350, 28.9872, 1.4094, 0.0094)
θ_0, θ_1, θ_2	0.178	0.236	0.381	B_3(3.5304, 2.1937, 29.8592, 1.000, 0.01)
		for Anderson-Darling test		
θ_0	1.899	2.458	3.853	B_3(2.7055, 3.0084, 16.8946, 12.5483, 0.12)
θ_1	1.771	2.338	3.729	B_3(2.6333, 2.6314, 22.5692, 12.5941, 0.125)
θ_2	1.188	1.467	2.118	B_3(2.7800, 5.1280, 11.7638, 10.5031, 0.11)
θ_0, θ_1	1.687	2.255	3.648	B_3(2.0354, 2.3209, 23.5136, 12.7679, 0.132)
θ_0, θ_2	1.153	1.432	2.090	B_3(3.6594, 3.4364, 13.5600, 5 .9140, 0.084)
θ_1, θ_2	0.985	1.239	1.862	B_3(4.0113, 3.4057, 19.6395, 6.2684, 0.084)
θ_0, θ_1, θ_2	0.886	1.143	1.785	B_3(4.1564, 2.7774, 30.5627, 6.0165, 0.0822)

Table 8

Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 5$)

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Kolmogorov test		
θ_0	1.223	1.357	1.626	B_3(3.4549, 6.2388, 2.0813, 2.300, 0.3600)
θ_1	1.205	1.342	1.613	B_3(3.1581, 6.2159, 1.9964, 2.300, 0.360)
θ_2	1.097	1.212	1.443	B_3(4.8171, 5.5295, 3.0757, 2.000, 0.320)
θ_0, θ_1	1.179	1.318	1.593	B_3(3.7224, 4.6425, 3.1224, 2.2000, 0.330)
θ_0, θ_2	1.097	1.213	1.444	B_3(4.9052, 5.6639, 2.9616, 2.0000, 0.310)
θ_1, θ_2	1.080	1.196	1.429	B_3(4.5122, 5.6639, 2.8588, 2.0000, 0.310)
θ_0, θ_1, θ_2	1.057	1.176	1.414	B_3(3.5446, 6.6218, 2.5197, 2.2850, 0.325)

(*continued*)

Table 8
Continued

Parameter estimated	Percentage points			Model
	0.9	0.95	0.99	
		for Cramer-Von Mises-Smirnov test		
θ_0	0.347	0.460	0.742	B_3(3.3548, 1.7217, 20.2585, 1.1000, 0.012)
θ_1	0.334	0.448	0.731	B_3(3.2927, 1.6388, 23.4040, 1.100, 0.012)
θ_2	0.236	0.303	0.469	B_3(4.0012, 2.0310, 17.0057, 0.730, 0.0095)
θ_0, θ_1	0.321	0.436	0.719	B_3(4.0952, 1.3628, 33.3948, 0.900, 0.0095)
θ_0, θ_2	0.236	0.304	0.470	B_3(3.8227, 2.0270, 16.0637, 0.7200, 0.0095)
θ_1, θ_2	0.219	0.287	0.453	B_3(4.1888, 1.9896, 21.3460, 0.7450, 0.009)
θ_0, θ_1, θ_2	0.206	0.275	0.444	B_3(4.5253, 1.7162, 31.4699, 0.715, 0.009)
		for Anderson-Darling test		
θ_0	1.908	2.467	3.861	B_3(3.2750, 2.7257, 19.7022, 11.000, 0.105)
θ_1	1.782	2.349	3.740	B_3(3.7185, 2.2262, 24.9194, 8.500, 0.10)
θ_2	1.292	1.608	2.358	B_3(3.8528, 2.9989, 12.7999, 5.2000, 0.09)
θ_0, θ_1	1.696	2.264	3.658	B_3(3.9441, 1.9099, 34.2183, 8.000, 0.085)
θ_0, θ_2	1.271	1.588	2.346	B_3(3.6684, 3.0110, 13.4931, 5.550, 0.085)
θ_1, θ_2	1.110	1.411	2.153	B_3(4.1345, 3.0883, 22.0926, 6.800, 0.080)
θ_0, θ_1, θ_2	1.014	1.322	2.084	B_3(4.3601, 2.6164, 38.0670, 7.4729, 0.0785)

Table 9
Upper percentage points and models of limiting statistic distributions of nonparametric goodness-of-fit test when using MLE (for $\theta_0 = 7$)

Parameter estimated	Percentage points			Model
	0.9	0.95	0.99	
		for Kolmogorov test		
θ_0	1.222	1.357	1.625	B_3(3.4527, 6.2874, 2.062, 2.300, 0.3600)
θ_1	1.210	1.345	1.616	B_3(3.1789, 6.3997, 1.9239, 2.300, 0.360)
θ_2	1.137	1.137	1.502	B_3(4.4660, 5.47624, 2.6851, 2.000, 0.320)
θ_0, θ_1	1.190	1.328	1.601	B_3(3.8325, 4.7340, 3.0569, 2.2000, 0.330)
θ_0, θ_2	1.137	1.259	1.503	B_3(4.9890, 5.1511, 3.1470, 2.0000, 0.310)
θ_1, θ_2	1.124	1.247	1.493	B_3(4.5766, 5.2588, 2.9181, 2.0000, 0.310)
θ_0, θ_1, θ_2	1.107	1.232	1.480	B_3(3.5462, 6.6218, 2.2864, 2.2850, 0.325)
		for Cramer-Von Mises-Smirnov test		
θ_0	0.347	0.460	0.742	B_3(3.4065, 1.72170, 20.5769, 1.1000, 0.012)
θ_1	0.336	0.451	0.733	B_3(3.3961, 1.6388, 23.7205, 1.100, 0.012)
θ_2	0.265	0.345	0.542	B_3(4.0337, 1.7885, 18.1049, 0.730, 0.0095)
θ_0, θ_1	0.325	0.440	0.722	B_3(4.5574, 1.36280, 36.1643, 0.900, 0.0095)
θ_0, θ_2	0.266	0.346	0.545	B_3(4.1853, 1.7329, 19.4044, 0.7200, 0.0095)
θ_1, θ_2	0.253	0.333	0.531	B_3(4.3597, 1.7257, 23.2817, 0.7450, 0.009)
θ_0, θ_1, θ_2	0.241	0.323	0.522	B_3(4.3835, 1.5744, 28.6719, 0.715, 0.009)

(*continued*)

Table 9
Continued

Parameter estimated	Percentage points 0.9	0.95	0.99	Model
		for Anderson-Darling test		
θ_0	1.916	2.475	3.864	B_3(3.3337, 2.7380, 19.7773, 11.000, 0.105)
θ_1	1.800	2.366	3.755	B_3(3.7496, 2.2445, 24.3153, 8.500, 0.10)
θ_2	1.431	1.804	2.698	B_3(3.9524, 2.6173, 14.0679, 5.2000, 0.09)
θ_0, θ_1	1.714	2.282	3.674	B_3(4.4259,1.8843, 34.1400, 7.30, 0.085)
θ_0, θ_2	1.419	1.793	2.697	B_3(3.6688, 2.7003, 13.7324, 5.550, 0.085)
θ_1, θ_2	1.279	1.644	2.539	B_3(4.1773, 2.7020, 22.9667, 6.800, 0.080)
θ_0, θ_1, θ_2	1.188	1.560	2.470	B_3(4.5480, 2.1191, 28.5121, 5.000, 0.0785)

The distributions $G(S \mid H_0)$ of the Kolmogorov, Cramer-Von Mises-Smirnov, and the Anderson-Darling statistics are best approximated by the family of the III type beta-distributions with the density function

$$B_3(\theta_0, \theta_1, \theta_2, \theta_3, \theta_4) = \frac{\theta_2^{\theta_0}}{\theta_3 \mathrm{B}(\theta_0, \theta_1)} \frac{\left(\frac{x-\theta_4}{\theta_3}\right)^{\theta_0-1}\left(1-\frac{x-\theta_4}{\theta_3}\right)^{\theta_1-1}}{\left[1+(\theta_2-1)\frac{x-\theta_4}{\theta_3}\right]^{\theta_0+\theta_1}},$$

by the gamma-distribution family

$$\gamma(\theta_0, \theta_1, \theta_2) = \frac{1}{\theta_1^{\theta_0}\Gamma(\theta_0)}(x-\theta_2)^{\theta_0-1}e^{-(x-\theta_2)\theta_1},$$

by the family of the *Sb*-Johnson distributions

$$Sb(\theta_0, \theta_1, \theta_2, \theta_3) = \frac{\theta_1\theta_2}{(x-\theta_3)(\theta_2+\theta_3-x)} \exp\left\{-\frac{1}{2}\left[\theta_0 - \theta_1 \ln\frac{x-\theta_3}{\theta_2+\theta_3-x}\right]^2\right\}$$

or by the family of the *Sl*-Johnson distributions

$$Sl(\theta_0, \theta_1, \theta_2, \theta_3) = \frac{\theta_1}{(x-\theta_3)\sqrt{2\pi}} \exp\left\{-\frac{1}{2}\left[\theta_0 + \theta_1 \ln\frac{x-\theta_3}{\theta_2}\right]^2\right\}.$$

The tables of the percentage points and the statistic distribution models were constructed by the simulated statistic samples with the size $N = 10^6$ (N is a number of runs in simulation). This number ensures that the deviation of the empirical pdf $G_N(S \mid H_0)$ from the theoretical (true) one will be less than 10^{-3}. In this case, the samples of the pseudorandom variables, belonging to $F(x, \theta)$, were generated with the size $n = 10^3$. With this value of n the statistic pdf $G(S_n \mid H_0)$ almost coincides with the limiting pdf $G(S \mid H_0)$.

3. Conclusions

The more accurate models of the statistic distributions of the nonparametric goodness-of-fit tests for testing composite hypotheses with the distribution family (4) were presented. The constructed models and the percentage points can be used in of statistical analysis practice when testing composite hypotheses of fitness for the distribution family (4).

It should be stressed, that the percentage points and the models obtained guarantee the proper implementation of the nonparametric goodness-of-fit tests in solving statistic analysis problems provided MLE is used. These results can't be used with other estimations because statistic distributions of these tests essentially depend on the estimation method used (Lemeshko and Postovalov, 2001b).

In the case of the I, II, III type beta-distribution families the statistic distributions depend on the specific values of two form parameters of these distributions. The statistic distributions models and the tables of percentage points for various combinations of values of two form parameters (more than 1,500 models) were constructed in the thesis by Lemeshko (2007) and partly were published in Lemeshko and Lemeshko (2007).

It should be noted that in composite hypotheses testing, the power of the nonparametric goodness-of-fit tests, generally, is essentially higher (if MLE is used), than that in simple hypotheses testing.

The results of the comparative analysis of the goodness-of-fit test power (nonparametric and χ^2 type) depending on some sufficiently close pairs of alternatives are presented in Lemeshko et al. (2007), and are stated in more detail in Lemeshko et al. (2009, 2010a).

The authors hope that the article will contribute to reducing the number of errors, while solving statistic analysis problems provided nonparametric goodness-of-fit tests are used (Lemeshko, 2004).

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