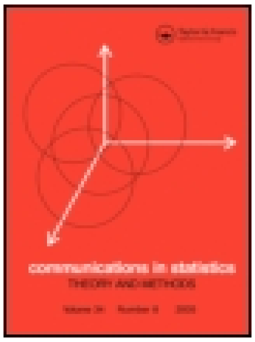

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Goodness-of-Fit Tests

Statistic Distribution Models for Some Nonparametric Goodness-of-Fit Tests in Testing Composite Hypotheses

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The results (tables of percentage points and statistic distribution models) for the Kolmogorov, Cramer–Von Mises–Smirnov, and Anderson–Darling tests, when unknown parameters are estimated by their MLEs, are presented in this article.

Keywords Anderson–Darling test; Composite hypotheses testing; Cramer–Von Mises–Smirnov test; Goodness-of-fit test; Kolmogorov test.

Mathematics Subject Classification 62G10.

1. Introduction

In composite hypothesis testing of the form $H_0 : F(x) \in \{F(x, \theta), \theta \in \Theta\}$, when the estimate $\hat{\theta}$ of the scalar or vector distribution parameter $F(x, \theta)$ is calculated by the same sample, nonparametric goodness-of-fit Kolmogorov, ω^2 Cramer–Von Mises–Smirnov, and Ω^2 Anderson–Darling tests lose the free distribution property.

The value

$$D_n = \sup_{|x|<\infty} |F_n(x) - F(x, \theta)|,$$

where $F_n(x)$ is the empirical distribution function and n is the sample size, is used in Kolmogorov test as a distance between the empirical and theoretical laws. In testing hypotheses, a statistic with Bolshev (Bolshev and Smirnov, 1983) correction

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of the form

$$S_K = \frac{(6nD_n + 1)}{6\sqrt{n}}, \tag{1}$$

where $D_n = \max(D_n^+, D_n^-)$,

$$D_n^+ = \max_{1 \le i \le n} \left\{ \frac{i}{n} - F(x_i, \theta) \right\}, \quad D_n^- = \max_{1 \le i \le n} \left\{ F(x_i, \theta) - \frac{i-1}{n} \right\},$$

n is the sample size, $x_1, x_2, \ldots, x_n$ are sample values in increasing order is usually used. The limit probability distribution function (pdf) of test (1) in testing simple hypotheses follows the Kolmogorov's pdf $K(S) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2 s^2}$.

In ω^2 Cramer–Mises–Smirnov test, one uses a statistic of the form

$$S_\omega = n\omega_n^2 = \frac{1}{12n} + \sum_{i=1}^{n} \left\{ F(x_i, \theta) - \frac{2i-1}{2n} \right\}^2, \tag{2}$$

and in test of Ω^2 Anderson–Darling type, the statistic of the form

$$S_\Omega = -n - 2\sum_{i=1}^{n} \left\{ \frac{2i-1}{2n} \ln F(x_i, \theta) + \left(1 - \frac{2i-1}{2n}\right) \ln(1 - F(x_i, \theta)) \right\}. \tag{3}$$

In testing a simple hypothesis, statistic (2) follows the pdf (Bolshev and Smirnov, 1983) of the form

$$a1(S) = \frac{1}{\sqrt{2s}} \sum_{j=0}^{\infty} \frac{\Gamma(j+1/2)\sqrt{4j+1}}{\Gamma(1/2)\Gamma(j+1)} \exp\left\{ -\frac{(4j+1)^2}{16S} \right\} \times \left\{ I_{-\frac{1}{4}}\left[\frac{(4j+1)^2}{16S} \right] - I_{\frac{1}{4}}\left[\frac{(4j+1)^2}{16S} \right] \right\},$$

where $I_{-\frac{1}{4}}(\cdot)$, $I_{\frac{1}{4}}(\cdot)$ – modified Bessel function

$$I_v(z) = \sum_{k=0}^{\infty} \frac{\left(\frac{z}{2}\right)^{v+2k}}{\Gamma(k+1)\Gamma(k+v+1)}, \quad |z| < \infty, \quad |\arg z| < \pi,$$

and statistic (3) follows the pdf (Bolshev and Smirnov, 1983) of the form

$$a2(S) = \frac{\sqrt{2\pi}}{S} \sum_{j=0}^{\infty} (-1)^j \frac{\Gamma\left(j+\frac{1}{2}\right)(4j+1)}{\Gamma\left(\frac{1}{2}\right)\Gamma(j+1)} \exp\left\{ -\frac{(4j+1)^2\pi^2}{8S} \right\} \times \int_0^\infty \exp\left\{ \frac{S}{8(y^2+1)} - \frac{(4j+1)^2\pi^2 y^2}{8S} \right\} dy.$$

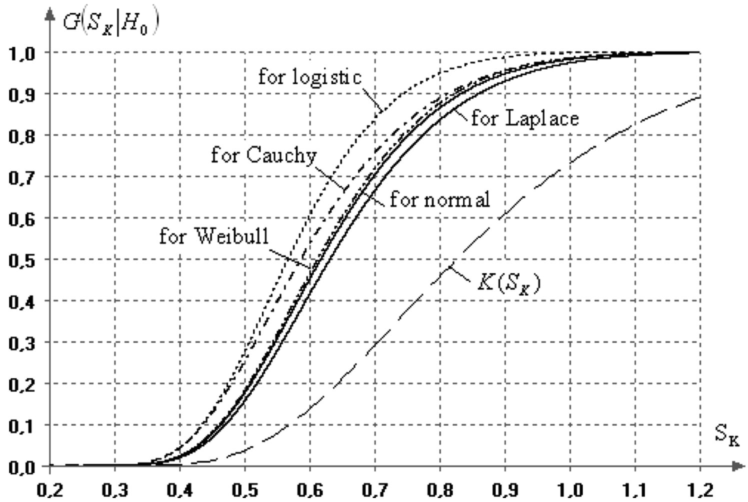

Figure 1. The Kolmogorov statistic (1) pdf's for testing composite hypotheses with calculating MLE of two law parameters.

2. Statistic Distributions of the Tests in Testing Composite Hypotheses

In composite hypotheses testing, the conditional distribution law of the statistic $G(S \mid H_0)$ is affected by a number of factors: the form of the observed law $F(x, \theta)$ corresponding to the true hypothesis H_0; the type of the parameter estimated and the number of parameters to be estimated; sometimes it is a specific value of the parameter (e.g., in the case of gamma-distribution and beta-distribution families); the method of parameter estimation. The distinctions in the limiting distributions of

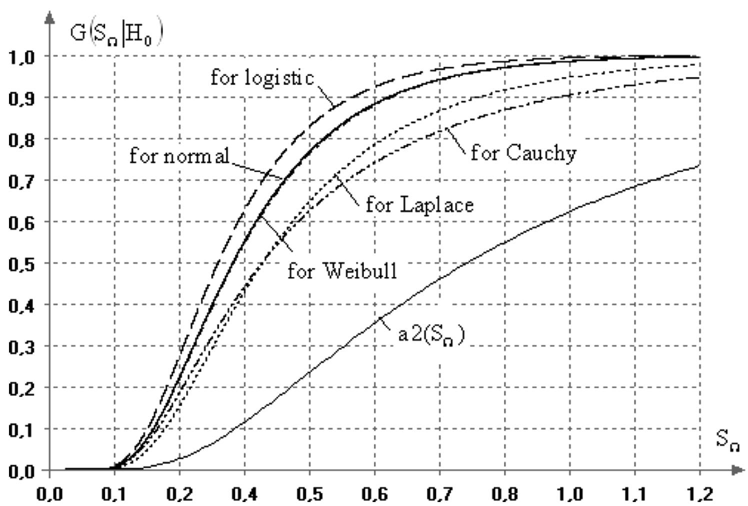

Figure 2. The Anderson–Darling statistic (3) pdf's for testing composite hypotheses with calculating MLE of two law parameters.

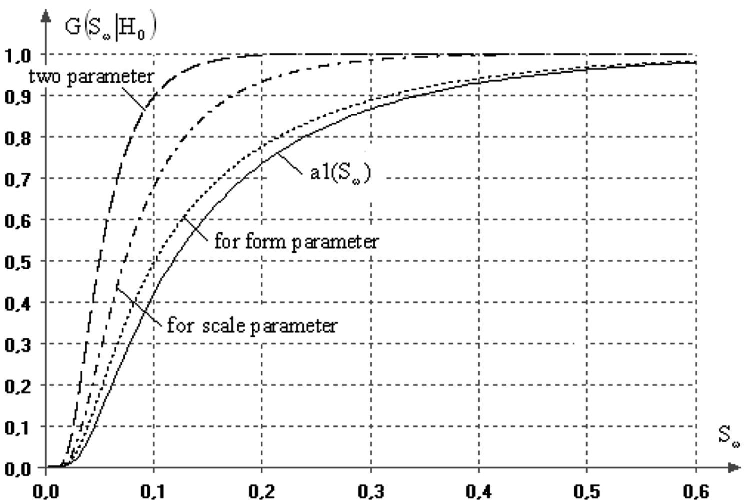

Figure 3. The Cramer–Mises–Smirnov statistic (2) pdf's for testing composite hypotheses with calculating MLE of Weibull distribution law parameters.

the same statistics in testing simple and composite hypotheses are so significant that we cannot neglect them. For example, Fig. 1 shows pdf's of the statistic (1) when testing different composite null hypotheses using maximum likelihood estimators (MLEs) of two parameters, and Fig. 2 represents pdf's of the statistic (3) in similar situation.

Figure 3 illustrates statistic distribution dependence (2) of the Cramer–Mises–Smirnov test upon the type of parameter estimated by the example of Weibull law.

Kac et al. (1955) was a pioneer in investigating statistic distributions of the nonparametric goodness-of-fit tests with composite hypotheses. Then, for the solution to this problem, various approaches were used (Chandra et al., 1981; Durbin, 1976; Martinov, 1978; Pearson and Hartley, 1972; Stephens, 1970, 1974; Tyurin, 1984; Tyurin and Savvushkina, 1984).

Table 1
Random variable distribution

Random variable distribution	Density function $f(x, \theta)$	Random variable distribution	Density function $f(x, \theta)$
Exponential	$\frac{1}{\theta_0} e^{-x/\theta_0}$	Laplace	$\frac{1}{2\theta_0} e^{-\lvert x-\theta_1 \rvert/\theta_0}$
Seminormal	$\frac{2}{\theta_0\sqrt{2\pi}} e^{-x^2/2\theta_0^2}$	Normal	$\frac{1}{\theta_0\sqrt{2\pi}} e^{-\frac{(x-\theta_1)^2}{2\theta_0^2}}$
Rayleigh	$\frac{x}{\theta_0^2} e^{-x^2/2\theta_0^2}$	Log-normal	$\frac{1}{x\theta_0\sqrt{2\pi}} e^{-(\ln x-\theta_1)^2/2\theta_0^2}$
Maxwell	$\frac{2x^2}{\theta_0^3\sqrt{2\pi}} e^{-x^2/2\theta_0^2}$	Cauchy	$\frac{\theta_0}{\pi[\theta_0^2+(x-\theta_1)^2]}$
	Density function $f(x, \theta)$		
Logistic	$\frac{\pi}{\theta_0\sqrt{3}} \exp\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\}/[1+\exp\{-\frac{\pi(x-\theta_1)}{\theta_0\sqrt{3}}\}]^2$		
Extreme-value (maximum)	$\frac{1}{\theta_0} \exp\{-\frac{x-\theta_1}{\theta_0} - \exp(-\frac{x-\theta_1}{\theta_0})\}$		
Extreme-value (minimum)	$\frac{1}{\theta_0} \exp\{\frac{x-\theta_1}{\theta_0} - \exp(\frac{x-\theta_1}{\theta_0})\}$		
Weibull	$\frac{\theta_0 x^{\theta_0-1}}{\theta_1^{\theta_0}} \exp\{-(\frac{x}{\theta_1})^{\theta_0}\}$		

Table 2
Upper percentage points of statistic distribution of the nonparametric goodness-of-fit tests for the case of MLE usage

Random variable distribution	Parameter estimated	Kolmogorov's test			Cramer–Von Mises–Smirnov's test			Anderson–Darling's test		
		0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
Exponential & Rayleigh	Scale	0.9946	1.0936	1.2919	0.1743	0.2214	0.3369	1.0599	1.3193	1.9537
Seminormal	Scale	1.0514	1.1599	1.3811	0.2054	0.2659	0.4151	1.1882	1.4987	2.2666
Maxwell	Scale	0.9687	1.0615	1.2511	0.1620	0.2040	0.3063	1.0095	1.2474	1.8318
Laplace	Scale	1.1770	1.3125	1.5858	0.3230	0.4378	0.7189	1.7260	2.2861	3.6836
	Shift	0.9565	1.0444	1.2225	0.1513	0.1865	0.2671	1.0698	1.3005	1.8322
	Two parameters	0.8629	0.9398	1.0960	0.1152	0.1437	0.2136	0.7970	0.9818	1.4404
Normal & Log-normal	Scale	1.1908	1.3274	1.5999	0.3273	0.4425	0.7265	1.7450	2.3089	3.7061
	Shift	0.8881	0.9632	1.1136	0.1344	0.1654	0.2377	0.8923	1.0872	1.5510
	Two parameters	0.8352	0.9086	1.0566	0.1034	0.1257	0.1777	0.6293	0.7497	1.0297
Cauchy	Scale	1.1372	1.2748	1.5503	0.3155	0.4300	0.7113	1.7157	2.2769	3.6728
	Shift	0.9753	1.0700	1.2603	0.1722	0.2162	0.3185	1.2154	1.5121	2.2114
	Two parameters	0.8151	0.8926	1.0478	0.1287	0.1699	0.2708	0.9480	1.2260	1.9129
Logistic	Scale	1.1797	1.3158	1.5893	0.3232	0.4380	0.7190	1.7244	2.2845	3.6820
	Shift	0.8371	0.9072	1.0464	0.1191	0.1476	0.2163	0.8559	1.0427	1.4951
	Two parameters	0.7465	0.8054	0.9230	0.0813	0.0976	0.1352	0.5619	0.6645	0.9027
Extreme-value & Weibull	Scale[1)]	1.1823	1.3157	1.5832	0.3201	0.4311	0.7043	1.7234	2.2726	3.6343
	Shift[2)]	0.9949	1.0931	1.2922	0.1742	0.2212	0.3364	1.0591	1.3183	1.9522
	Two parameters	0.8243	0.8948	1.0366	0.1017	0.1235	0.1744	0.6336	0.7548	1.0396

Note. [1)]we estimated the Weibull distribution shape parameter; [2)]the Weibull distribution scale parameter.

Table 3
Models of limiting statistic distributions of nonparametric goodness-of-fit when MLE are used

Test	Random variable distribution	Estimation of scale parameter	Estimation of shift parameter	Estimation of two parameters
Kolmogorov's	Exponential & Rayleigh	γ(5.1092; 0.0861; 0.2950)	–	–
	Seminormal	γ(4.5462; 0.1001; 0.3100)	–	–
	Maxwell	γ(5.4566; 0.0794; 0.2870)	–	–
	Laplace	γ(3.3950; 0.1426; 0.3405)	γ(5.1092; 0.0861; 0.2950)	γ(6.2949; 0.0624; 0.2613)
	Normal & Log-normal	γ(3.5609; 0.1401; 0.3375)	γ(7.5304; 0.0580; 0.2400)	γ(6.4721; 0.0580; 0.2620)
	Cauchy	γ(3.0987; 0.1463; 0.3350)	γ(5.9860; 0.0780; 0.2528)	γ(5.3642; 0.0654; 0.2600)
	Logistic	γ(3.4954; 0.1411; 0.3325)	γ(7.6325; 0.0531; 0.2368)	γ(7.5402; 0.0451; 0.2422)
	Extreme-value & Weibull	γ(3.6805; 0.1355; 0.3350)[1)]	γ(5.2194; 0.0848; 0.2920)[2)]	γ(6.6012; 0.0563; 0.2598)
Cramer–Von Mises–Smirnov's	Exponential & Rayleigh	*Sb*(3.3738; 1.2145; 1.0792; 0.011)	–	–
	Seminormal	*Sb*(3.527; 1.1515; 1.5527; 0.012)	–	–
	Maxwell	*Sb*(3.353; 1.220; 0.9786; 0.0118)	–	–
	Laplace	*Sb*(3.2262; 0.9416; 2.703; 0.015)	*Sb*(2.9669; 1.2534; 0.6936; 0.01)	*Sb*(3.768; 1.2865; 0.8336; 0.0113)
	Normal & Log-normal	*Sb*(3.153; 0.9448; 2.5477; 0.016)	*Sb*(3.243; 1.315; 0.6826; 0.0095)	*Sb*(4.3950; 1.4428; 0.915; 0.009)
	Cauchy	*Sb*(3.1895; 0.9134; 2.690; 0.013)	*Sb*((2.359; 1.0732; 0.595; 0.0129)	*Sb*(3.4364; 1.0678; 1.000; 0.011)
	Logistic	*Sb*(3.264; 0.9581; 2.7046; 0.014)	*Sb*(4.0026; 1.2853; 1.00; 0.0122)	*Sb*(3.2137; 1.3612; 0.36; 0.0105)
	Extreme-value & Weibull	*Sb*(3.343; 0.9817; 2.753; 0.015)[1)]	*Sb*(3.498; 1.2236; 1.1632; 0.01)[2)]	*Sb*(3.3854; 1.4453; 0.4986; 0.007)
Anderson–Darling's	Exponential & Rayleigh	*Sb*(3.8386; 1.3429; 7.500; 0.090)	–	–
	Seminormal	*Sb*(4.2019; 1.2918; 11.500; 0.100)	–	–
	Maxwell	*Sb*(3.9591; 1.3296; 7.800; 0.1010)	–	–
	Laplace	*Sb*(4.3260; 1.0982; 27.00; 0.110)	*Sb*(3.1506; 1.3352; 4.9573; 0.096)	*Sb*(3,8071; 1,3531; 5,1809; 0,10)
	Normal & Log-normal	*Sb*(4.3271; 1.0895; 28.000; 0.120)	*Sb*(3.3085; 1.4043; 4.2537; 0.080)	*Sb*(3.5601; 1.4846; 3.0987; 0.08)
	Cauchy	*Sb*(3.7830; 1.0678; 18.0; 0.11)	*Sb*(3.4814; 1.2375; 7.810; 0.1)	*Sb*(3.290; 1.129; 5.837; 0.099)
	Logistic	*Sb*(3.516; 1.054; 14.748; 0.117)	*Sb*(5.1316; 1.5681; 10.0; 0.065)	*Sb*(3.409; 1.434; 2.448; 0.095)
	Extreme-value & Weibull	*Sb*(3.512; 1.064; 14.496; 0.125)[1)]	*Sb*(4.799; 1.402; 13.0; 0.085)[2)]	*Sb*(3.4830; 1.5138; 3.00; 0.07)

Note. [1)]we estimated the Weibull distribution shape parameter; [2)]the Weibull distribution scale parameter.

In our research (Lemeshko and Postovalov, 1998, 2001; Lemeshko and Maklakov, 2004), statistic distributions of the nonparametric goodness-of-fit tests are investigated by the methods of statistical simulating, and for constructed empirical distributions approximate models of law are found. The results obtained were used by us to develop recommendations for standardization (R 50.1.037-2002, 2002).

3. Improvement of Statistic Distribution Models of the Nonparametric Goodness-of-Fit Tests

In this article, we present more precise results (tables of percentage points and statistic distribution models) for the nonparametric goodness-of-fit tests in testing composite hypotheses using the MLE. Table 1 contains a list of distributions relative to which we can test composite fit hypotheses using the constructed approximations of the limiting statistic distributions.

Upper percentage points are presented in Table 2, and constructed statistic distribution models are presented in Table 3. Percentage points in Table 2 and models in Table 3 do not depend on values of unknown parameters of the law $F(x, \theta)$.

Distributions $G(S \mid H_0)$ of the Kolmogorov statistic are best approximated by gamma-distributions family (see Table 3) with the density function

$$\gamma(\theta_0, \theta_1, \theta_2) = \frac{1}{\theta_1^{\theta_0}\Gamma(\theta_0)}(x-\theta_2)^{\theta_0-1}e^{-(x-\theta_2)/\theta_1},$$

and distributions of the Cramer–Mises–Smirnov and Anderson–Darling statistics are well approximated by the family of the *Sb*-Johnson distributions with the density function

$$Sb(\theta) = \frac{\theta_1\theta_2}{(x-\theta_3)(\theta_2+\theta_3-x)}\exp\left\{-\frac{1}{2}\left[\theta_0 - \theta_1 \ln\frac{x-\theta_3}{\theta_2+\theta_3-x}\right]^2\right\}.$$

The tables of percentage points and statistic distributions models were constructed by modeled statistic samples with the size $N = 10^6$ (N is the number of runs in simulation). This number ensures the deviation of the empirical pdf $G_N(S \mid H_0)$ from the theoretical (true) to be less than 10^{-3}. In this case, the samples of pseudorandom variables, belonging to $F(x, \theta)$, were generated with the size $n = 10^3$. For such value of n, statistic pdf $G(S_n \mid H_0)$ almost coincides with the limit pdf $G(S \mid H_0)$. An accuracy of constructed percentage points can be indirectly assessed by the closeness of obtained results to the known results. For example, for $\alpha = 0.1, 0.05, 0.01$ percentage points obtained by the analytical methods (see Martinov, 1978) for testing null hypothesis about the normal law when two parameters are unknown, are equal to 0.1035, 0.1260, 0.1788, correspondingly. In this article (see Table 2), we have obtained: 0.1034, 0.1257, 0.1777, correspondingly.

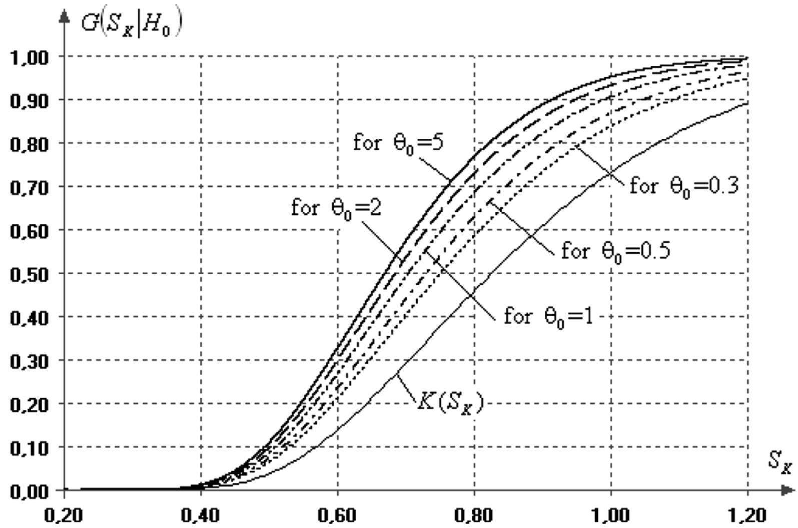

Figure 4. The Kolmogorov statistic (1) distributions for testing composite hypotheses with calculating MLE of scale parameter depend on the shape parameter value of gamma-distribution.

4. Improvement of Statistic Distribution Models of the Nonparametric Goodness-of-Fit Tests in the Case of Gamma Distribution

In composite hypotheses testing subject to gamma distribution with the density function

$$f(x, \theta) = \frac{x^{\theta_0 - 1}}{\theta_1^{\theta_0}\Gamma(\theta_0)} \exp\left(-\frac{x}{\theta_1}\right)$$

limiting statistics distributions of the nonparametric goodness-of-fit tests depend on values of the shape parameter θ_0. For example, Fig. 4 illustrates dependence of the Kolmogorov statistic distribution upon the value θ_0 in testing a composite hypothesis in the case of calculating MLE for the scale parameter of gamma-distribution only.

Upper percentage points constructed as a result of statistical modeling are presented in Table 4, and statistics distributions models are given in Table 5. In this case, statistics distributions are well approximated by the family of the III.

Type beta-distributions with the density function

$$B_3(\theta_0, \theta_1, \theta_2, \theta_3, \theta_4) = \frac{\theta_2^{\theta_0}}{\theta_3 B(\theta_0, \theta_1)} \frac{\left(\frac{x-\theta_4}{\theta_3}\right)^{\theta_0 - 1}\left(1 - \frac{x-\theta_4}{\theta_3}\right)^{\theta_1 - 1}}{\left[1 + (\theta_2 - 1)\frac{x-\theta_4}{\theta_3}\right]^{\theta_0 + \theta_1}}.$$

The results presented in R 50.1.037-2002 (2002) are made considerably more precise by the upper percentage points and statistics distributions models given in Tables 4 and 5.

Table 4
Upper percentage points of statistic distribution of the nonparametric goodness-of-fit tests when MLE are used in the case of gamma-distribution

Value of the shape parameter	Parameter estimated	Kolmogorov's test			Cramer–Von Mises–Smirnov's test			Anderson–Darling's test		
		0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
0.3	Scale	1.096	1.211	1.444	0.233	0.305	0.482	1.300	1.655	2.543
	Shape	0.976	1.070	1.262	0.166	0.209	0.316	1.021	1.258	1.865
	Two parameters	0.905	0.990	1.162	0.127	0.158	0.233	0.718	0.870	1.233
0.5	Scale	1.051	1.160	1.379	0.205	0.264	0.413	1.183	1.490	2.260
	Shape	0.961	1.052	1.236	0.159	0.199	0.298	0.993	1.221	1.791
	Two parameters	0.884	0.965	1.131	0.119	0.146	0.212	0.684	0.824	1.145
1.0	Scale	0.994	1.095	1.299	0.175	0.220	0.336	1.058	1.313	1.955
	Shape	0.936	1.022	1.191	0.149	0.186	0.273	0.952	1.166	1.696
	Two parameters	0.862	0.940	1.097	0.111	0.136	0.194	0.657	0.785	1.084
2.0	Scale	0.952	1.044	1.228	0.155	0.193	0.288	0.980	1.203	1.771
	Shape	0.915	0.995	1.155	0.142	0.176	0.256	0.922	1.125	1.625
	Two parameters	0.849	0.924	1.077	0.107	0.131	0.185	0.643	0.766	1.051
3.0	Scale	0.933	1.020	1.200	0.148	0.184	0.272	0.952	1.163	1.702
	Shape	0.906	0.985	1.140	0.139	0.172	0.251	0.912	1.110	1.601
	Two parameters	0.845	0.919	1.070	0.106	0.129	0.182	0.639	0.761	1.043
4.0	Scale	0.923	1.008	1.181	0.145	0.179	0.264	0.937	1.141	1.662
	Shape	0.901	0.980	1.132	0.138	0.171	0.248	0.906	1.103	1.590
	Two parameters	0.843	0.916	1.066	0.105	0.128	0.180	0.637	0.758	1.039
5.0	Scale	0.917	1.000	1.170	0.142	0.176	0.259	0.927	1.130	1.640
	Shape	0.899	0.977	1.127	0.137	0.169	0.246	0.902	1.099	1.586
	Two parameters	0.842	0.915	1.063	0.105	0.128	0.179	0.636	0.757	1.037

Table 5

Models of limiting statistic distributions of the nonparametric goodness-of-fit when MLE are used in the case of gamma-distribution

Test	Value of the shape parameter	Estimation of scale parameter	Estimation of shape parameter	Estimation of two parameters
Kolmogorov's	**0.3**	B_3(6.6871; 4.8368; 4.4047; 1.9440; 0.281)	B_3(6.4536; 5.7519; 3.3099; 1.6503; 0.280)	B_3(6.9705; 5.6777; 3.6297; 1.5070; 0.270)
	0.5	B_3(6.9356; 5.0081; 4.3582; 1.8470; 0.280)	B_3(6.3860; 5.9685; 3.1228; 1.6154; 0.280)	B_3(6.4083; 5.9339; 3.2063; 1.4483; 0.2774)
	1.0	B_3(6.7187; 5.3740; 3.7755; 1.6875; 0.282)	B_3(6.1176; 6.4704; 2.6933; 1.5501; 0.280)	B_3(5.6031; 6.1293; 2.7065; 1.3607; 0.2903)
	2.0	B_3(5.8359; 22.6032; 2.1921; 4.00; 0.282)	B_3(6.1387; 6.5644; 2.6021; 1.4840; 0.280)	B_3(5.8324; 6.1446; 2.7546; 1.3280; 0.2862)
	3.0	B_3(5.9055; 24.4312; 2.0996; 4.00; 0.282)	B_3(6.1221; 6.6131; 2.5536; 1.4590; 0.280)	B_3(6.0393; 6.1276; 2.8312; 1.3203; 0.2827)
	4.0	B_3(5.9419; 27.1264; 1.9151; 4.00; 0.282)	B_3(6.0827; 6.7095; 2.4956; 1.4494; 0.280)	B_3(6.1584; 6.1187; 2.8748; 1.3170; 0.2807)
	5.0	B_3(5.8774; 30.0692; 1.7199; 4.00; 0.282)	B_3(6.0887; 6.7265; 2.4894; 1.4432; 0.280)	B_3(6.1957; 6.1114; 2.8894; 1.3140; 0.2801)
Cramer–Von Mises–Smirnov's	**0.3**	B_3(3.2722; 1.9595; 16.1768; 0.750; 0.013)	B_3(3.0247; 3.2256; 11.113; 0.7755; 0.0125)	B_3(2.3607; 4.0840; 7.0606; 0.6189; 0.0145)
	0.5	B_3(3.2296; 2.1984; 14.3153; 0.700; 0.013)	B_3(3.0143; 3.3504; 10.095; 0.7214; 0.0125)	B_3(2.7216; 3.9844; 7.4993; 0.5372; 0.013)
	1.0	B_3(3.1201; 2.5460; 11.1200; 0.600; 0.013)	B_3(2.9928; 3.4716; 8.8275; 0.6346; 0.0125)	B_3(3.0000; 3.8959; 7.3247; 0.4508; 0.012)
	2.0	B_3(2.9463; 3.1124; 9.1160; 0.600; 0.013)	B_3(2.9909; 3.5333; 8.2010; 0.5786; 0.0125)	B_3(3.0533; 3.9402; 7.1173; 0.4246; 0.0118)
	3.0	B_3(2.8840; 3.3796; 8.4342; 0.600; 0.013)	B_3(2.9737; 3.5528; 7.8843; 0.5549; 0.0125)	B_3(3.0703; 3.9618; 7.034; 0.4163; 0.0117)
	4.0	B_3(2.8522; 3.5285; 8.1044; 0.600; 0.013)	B_3(2.9677; 3.5426; 7.7632; 0.5418; 0.0125)	B_3(3.0967; 3.9539; 7.064; 0.4122; 0.0116)
	5.0	B_3(2.8249; 3.6280; 7.8756; 0.6000; 0.013)	B_3(2.9638; 3.5465; 7.6558; 0.5334; 0.0125)	B_3(4.4332; 3.6256; 10.552; 0.4098; 0.0084)
Anderson–Darling's	**0.3**	B_3(3.3848; 2.8829; 14.684; 6.0416; 0.1088)	B_3(3.1073; 3.7039; 8.6717; 4.3439; 0.1120)	B_3(4.5322; 4.060; 10.0718; 2.9212; 0.078)
	0.5	B_3(5.0045; 2.9358; 18.8524; 5.2436; 0.077)	B_3(3.1104; 3.7292; 8.0678; 4.0132; 0.1120)	B_3(5.0079; 4.056; 10.0292; 2.5872; 0.073)
	1.0	B_3(5.0314; 3.1848; 15.4626; 4.3804; 0.077)	B_3(3.1149; 3.7919; 7.4813; 3.6770; 0.1120)	B_3(5.0034; 4.1093; 9.1610; 2.3427; 0.073)
	2.0	B_3(4.9479; 3.3747; 13.0426; 3.8304; 0.077)	B_3(3.0434; 4.1620; 7.1516; 3.8500; 0.1120)	B_3(4.9237; 4.2091; 8.6643; 2.2754; 0.073)
	3.0	B_3(5.0367; 3.4129; 12.9013; 3.6867; 0.077)	B_3(3.0565; 3.9092; 6.7844; 3.3972; 0.1120)	B_3(4.9475; 4.2070; 8.6686; 2.2512; 0.073)
	4.0	B_3(4.9432; 3.5038; 12.2240; 3.6302; 0.077)	B_3(3.0531; 3.9437; 6.7619; 3.3993; 0.1120)	B_3(4.9274; 4.2279; 8.5573; 2.2390; 0.073)
	5.0	B_3(4.8810; 3.5762; 11.7894; 3.6051; 0.077)	B_3(3.0502; 3.9640; 6.7510; 3.4024; 0.1120)	B_3(4.9207; 4.2432; 8.4881; 2.2314; 0.073)

5. Conclusions

In this article, we present more precise models of statistic distributions of the nonparametric goodness-of-fit tests in testing composite hypotheses subject to some laws considered in recommendations for standardization (R 50.1.037-2002, 2002). Models of statistic distributions of the nonparametric goodness-of-fit tests in composite hypothesis testing relative to *Sb*-, *Sl*-, *Su*-Johnson family distributions (Lemeshko and Postovalov, 2002) and exponential family (Lemeshko and Maklakov, 2004) were made more precise earlier and did not improve now.

In the case of the Types I, II, III beta-distribution families' statistic distributions depend on a specific value of two shape parameter of these distributions. Statistic distributions models and tables of percentage points for various combinations of values of two shape parameters (more than 1,500 models) were constructed in the dissertation (Lemeshko, 2007).

The results of comparative analysis of goodness-of-fit tests power (nonparametric and χ^2 type) subject to some sufficiently close pair of alternative are presented in Lemeshko et al. (2007).

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