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APPLICATION AND POWER OF CRITERIA FOR TESTING THE HOMOGENEITY OF VARIANCES. PART II. NONPARAMETRIC CRITERIA

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The distributions and power of nonparametric tests of the homogeneity of dispersion characteristics (Ansari–Bradley, Mood, Siegel–Tukey, Capon, and Klotz) are studied. A comparative analysis is made of their power relative to the classical tests for the homogeneity of variance (Fisher, Bartlett, Cochran, Hartley, Levene). Tables of percentage points for the Cochran test are given for non-normal distributions.

Key words: *nonparametric tests, Ansari–Bradley, Mood, Siegel–Tukey, Capon, and Klotz tests, power of tests.*

This paper is a continuation of an earlier paper [1]. Here we examine the nonparametric (rank) Ansari–Bradley [2], Mood [3], Siegel–Tukey [4], Capon [5], and Klotz [6] tests. These tests are intended for the testing of hypotheses regarding the homogeneity of the scale parameters of distributions corresponding to analyzed samples. As a rule, the dispersion characteristics and, therefore, the standard deviation σ are proportional to the scaling parameter of the distribution. Thus, these tests can be regarded as nonparametric analogs to the tests for the homogeneity of variance.

In the present case, only two samples are compared. Hence, the hypothesis regarding the homogeneity of variance to be tested will have the form

$$H_0 : \sigma_1^2 = \sigma_2^2, \tag{1}$$

and the competing hypothesis will be

$$H_1 : \sigma_1^2 \neq \sigma_2^2. \tag{2}$$

For comparing the powers of the nonparametric tests with the parametric tests, we shall examine the same competing hypotheses as in [1]: $H_1 : \sigma_2 = 1.1\sigma_1$; $H_2 : \sigma_2 = 1.2\sigma_1$; $H_3 : \sigma_2 = 1.5\sigma_1$, We shall also use the statistical model of [7] and the ISW (Interval Statistics for Windows) program system based on [8]. The sizes of the samples of the statistics to be modelled will, as before, be $N = 10^6$, in order to ensure that the absolute value of the difference between the true distribution law for the statistic and the empirically modelled value is at most 10^{-3}.

The distributions of the statistics were studied for different parametric models of their distributions, but for validity of the comparison with the results obtained earlier [1], in the present case the sample models used the same family of distributions with the density

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$$De(\theta_0) = f(x;\theta_0,\theta_1,\theta_2) = \frac{\theta_0}{2\theta_1\Gamma(1/\theta_0)}\exp\left(-\left(\frac{|x-\theta_2|}{\theta_1}\right)^{\theta_0}\right) \tag{3}$$

with different values of the shape factor θ_0. As special cases, the distribution $De(\theta_0)$ includes the Laplace (θ_0= 1) and the normal (θ_0 = 2) distributions.

The Ansari–Bradley Test. The nonparametric analogs of the tests of homogeneity of dispersion are intended for testing hypotheses about whether two samples of volume n_1 and n_2 belong to a common general set with the same dispersion characteristics. As a rule, here it is assumed that the means are equal.

Let $n_1 < n_2$. The statistic for the Ansari–Bradley test [2] can be calculated using the formula

$$S = \sum_{i=1}^{n_1}\left\{\frac{n_1+n_2+1}{2} - \left|R_i - \frac{n_1+n_2+1}{2}\right|\right\}, \tag{4}$$

where R_i are the ranks of the first (smaller in volume) sample in the common variational series.

The hypothesis to be tested is not rejected with probability α for $S_{\alpha/2} < S < S_{1-\alpha/2}$. The critical values of the statistic for n_1, $n_2 \le 10$ can be found, for example, in [9], and for larger n_i the table of values can be extended in elementary fashion by statistical modelling techniques.

In the case where the samples of random quantities have the same distribution $G(S|H_0)$ for the (4) statistic, when the test hypothesis is true H_0 does not depend on the form of this distribution. Thus, the mathematical expectation and dispersion of the statistic (4) are given by

$$E[S] = \begin{cases} n_1(n_1+n_2+2)/4 & \text{for even } (n_1+n_2); \\ n_1(n_1+n_2+1)^2/4(n_1+n_2) & \text{for odd } (n_1+n_2); \end{cases}$$

$$D[S] = \begin{cases} n_1n_2(n_1+n_2-2)(n_1+n_2+2)/48(n_1+n_2-1) & \text{for even } (n_1+n_2); \\ n_1n_2(n_1+n_2+1)[(n_1+n_2)^2+3]/48(n_1+n_2)^2 & \text{for odd } (n_1+n_2). \end{cases}$$

For sample volumes n_1, $n_2 > 10$, the discrete distribution of the normalized statistic

$$S^* = (S - E[S])/\sqrt{D[S]} \tag{5}$$

is approximated by a standard normal law. In this case, the test hypothesis is not rejected for $N^*_{\alpha/2} < S^* < N^*_{1-\alpha/2}$, where N^*_α is the corresponding quantile for a normal distribution. The discreteness of the distributions of the (4) and (5) statistics can essentially be neglected for n_1, $n_2 > 40$.

The distribution of the normalized statistic (5) is plotted in Fig. 1*a* for the cases in which the different competing hypotheses H_i hold for sample sizes $n_1 = n_2 = 10$ obeying the family (3) with a shape factor $\theta_0 = 3$.

The Mood Test. The statistic for this test is [3, 10]

$$M = \sum_{i=1}^{n_1}\left(R_i - \frac{n_1+n_2+1}{2}\right)^2, \tag{6}$$

where the R_i are the ranks of the samples of smallest size in the common variational series of the two samples.

The test hypothesis is not rejected for $M_{\alpha/2} < M < M_{1-\alpha/2}$. The critical values for this statistic for n_1, $n_2 \le 10$ can also be found in [9], while for larger n_i these values can easily be extended by statistical modelling.

For n_1, $n_2 > 10$, the distribution of the normalized statistic

$$M^* = (M - E[M] + 1/2)/\sqrt{D[M]}, \tag{7}$$

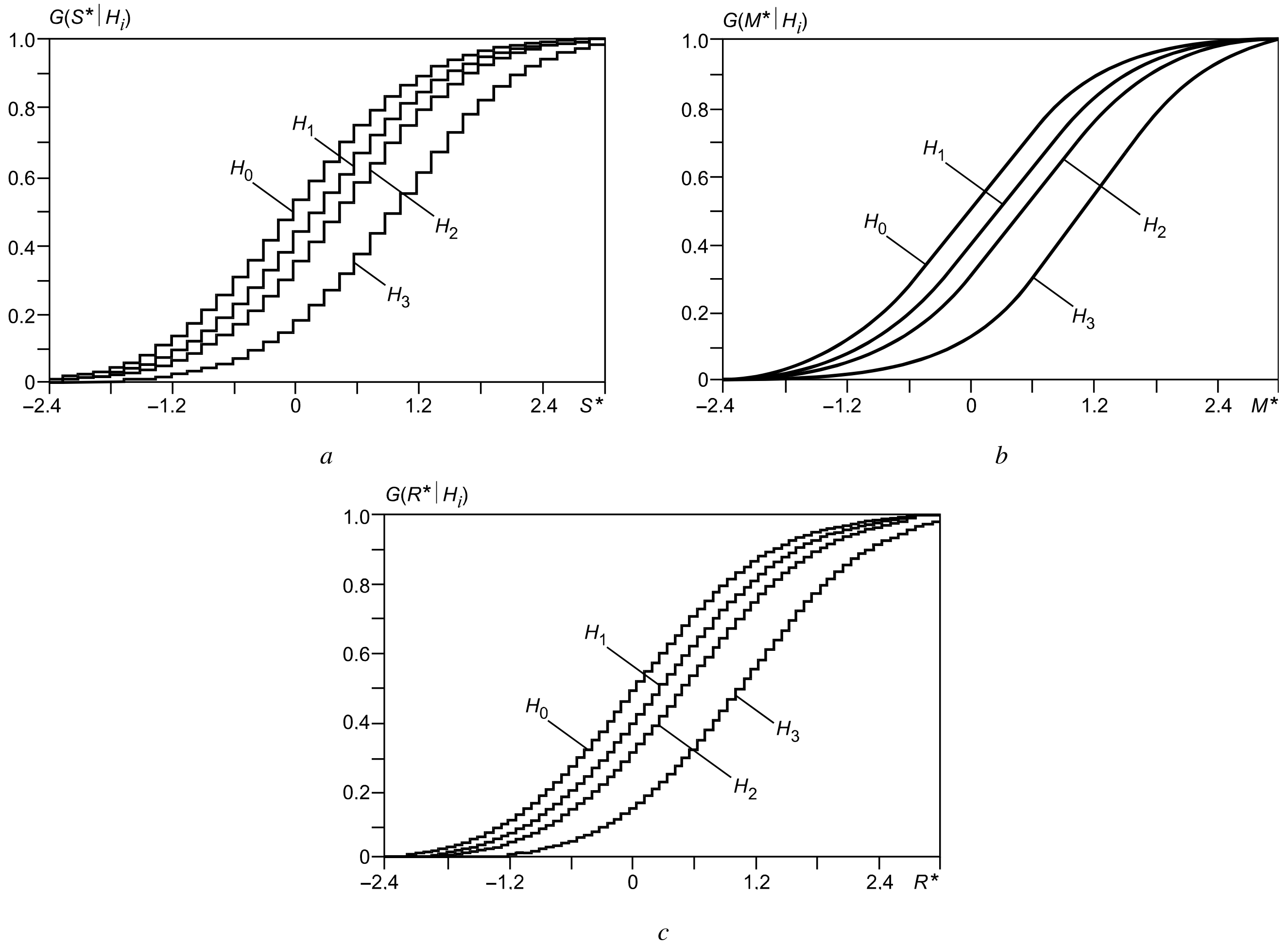

Fig. 1. Distributions of the normalized statistics for the Ansari–Bradley S^* (*a*), Mood M^* (*b*), and Siegel–Tukey R^* (*c*) tests; the samples with sizes $n_1 = n_2 = 10$ belong to the (3) distribution.

where

$$E[M] = n_1(n_1 + n_2 + 1)(n_1 + n_2 - 1)/12;$$

$$D[M] = n_1 n_2 (n_1 + n_2 + 1)(n_1 + n_2 + 2)(n_1 + n_2 - 2)/180,$$

is well approximated by a standard normal law [11], and for $n_1, n_2 > 20$, studies have shown that the discreteness of Eqs. (6) and (7) can be neglected entirely. When Eq. (7) is used, the test hypothesis is not rejected for $N^*_{\alpha/2} < M^* < N^*_{1-\alpha/2}$.

The distributions of the normalized statistic (7) are plotted in Fig. 1*b* for the cases in which the different competing hypotheses H_i hold with sample sizes $n_1 = n_2 = 10$ with adherence to the family of distributions (3) for a form factor $\theta_0 = 3$. It can be seen that the discreteness of this statistic is considerably less evident than for the Ansari–Bradley test (Fig. 1*a*).

The Siegel–Tukey Test. The variational series constructed from the combined sample $x_1 \le x_2 \le ... \le x_n$, where $n = n_1 + n_2$, is transformed into the sequence

$$x_1, x_n, x_{n-1}, x_2, x_3, x_{n-2}, x_{n-3}, x_4, x_5, ...,$$

i.e., the remaining series is "turned over" each time after attribution of the ranks of the pair of extreme values [4]. The statistic for this test is the sum of the ranks of the smaller volume sample. If $n_1 < n_2$, the statistic for this test is

$$R = \sum_{i=1}^{n_1} R_i . \tag{8}$$

The test hypothesis is not rejected for $R_{\alpha/2} < R < R_{1-\alpha/2}$. The (8) statistic is the analog of the Mann–Whitney test for testing hypotheses regarding the homogeneity of scale parameters. Thus, the quantiles of the Mann–Whitney test [9] for hypothesis testing. For n_1, $n_2 > 10$, the distribution of the normalized statistic

$$R^* = (R - E[R]) / \sqrt{D[R]}, \tag{9}$$

where

$$E[R] = n_1(n_1 + n_2 + 1)/2; \quad D[R] = n_1 n_2 (n_1 + n_2 + 1)/12$$

is fairly well approximated by a standard normal law. In this case, the test hypothesis is not rejected for $N^*_{\alpha/2} < R^* < N^*_{1-\alpha/2}$. Here the discreteness of the distribution of the statistic can essentially be neglected, beginning with sample sizes n_1, $n_2 > 30$.

Distributions of the normalized statistic (9) for the Siegel–Tukey test are plotted in Fig. 1*c* for the cases in which the different competing hypotheses hold for sample sizes $n_1 = n_2 = 10$ adhering to the family (3). Note that this statistic is more discrete than the statistic (7) (Fig. 1*b*) and is less so than the statistic (5) (Fig. 1*a*).

The Capon Test. The statistic for the Capon test [5] is

$$K = \sum_{i=1}^{n_1} a_{n_1+n_2}(R_i), \tag{10}$$

where $n_1 < n_2$; R_i is the rank of the *i*th element of the smallest sized sample in the common series in increasing order $(n_1 + n_2)$; and $a_{n_1+n_2}(i)$ is the mathematical expectation of the square of the *i*th order statistic in the sample of size $(n_1 + n_2)$ from a standard normal law. The markers $a_N(i)$ for this test are given in [9], for example.

The hypothesis that the scale parameters are equal is not rejected with a probability α if $K_{\alpha/2} < K < K_{1-\alpha/2}$, where the critical values $K_{\alpha/2}$ and $K_{1-\alpha/2}$ can also be found in [9].

For n_1, $n_2 > 10$, the normal approximation

$$K^* = (K - E[K]) / \sqrt{D[K]} \tag{11}$$

holds, where $E[K] = n_1$;

$$D[K] = \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 1)} \sum_{i=1}^{n_1+n_2} a^2_{n_1+n_2}(i) - \frac{n_1 n_2}{n_1 + n_2 - 1}.$$

The hypothesis of equal scale parameters is not rejected if $N^*_{\alpha/2} < K^* < N^*_{1-\alpha/2}$.

Unlike the previous tests, the distributions of the statistics in Eqs. (10) and (11) are quite smooth.

The Klotz Test. The statistic for this test is [6]

$$L = \sum_{i=1}^{n_1} u^2_{R_i/(n_1+n_2+1)} \tag{12}$$

where $(n_1 \le n_2)$; u_γ is the γ-quantile of the standard normal distribution; and the markers for this test, $u^2_{i/(N+1)}$, are given, for example, in [9].

TABLE 1. Power of Tests with Respect to the Competing Hypotheses $H_1 : \sigma_2 = 1.1\sigma_1$

Test	α	Sample size n				
		10	20	40	60	100
Mood	0.1	0.111	0.120	0.143	0.166	0.211
	0.05	0.057	0.064	0.080	0.096	0.128
	0.01	0.012	0.014	0.020	0.026	0.039
Ansari–Bradley	0.1	0.101	0.125	0.135	0.154	0.190
	0.05	0.052	0.064	0.074	0.087	0.113
	0.01	0.011	0.014	0.019	0.023	0.033
Siegel–Tukey	0.1	0.101	0.121	0.135	0.154	0.190
	0.05	0.055	0.062	0.075	0.087	0.113
	0.01	0.011	0.010	0.018	0.023	0.033

TABLE 2. Power of Tests with Respect to the Competing Hypotheses $H_2 : \sigma_2 = 1.2\sigma_1$

Test	α	Sample size n				
		10	20	40	60	100
Mood	0.1	0.135	0.173	0.254	0.330	0.468
	0.05	0.073	0.100	0.161	0.222	0.344
	0.01	0.016	0.027	0.052	0.082	0.152
Ansari–Bradley	0.1	0.128	0.172	0.226	0.289	0.406
	0.05	0.070	0.097	0.141	0.189	0.287
	0.01	0.015	0.025	0.045	0.066	0.119
Siegel–Tukey	0.1	0.124	0.165	0.226	0.289	0.405
	0.05	0.066	0.092	0.141	0.190	0.287
	0.01	0.014	0.013	0.044	0.066	0.119

The hypothesis of equal scale parameters is not rejected with probability α if $L_{\alpha/2} < L < L_{1-\alpha/2}$, where the critical values of $L_{\alpha/2}$ and $L_{1-\alpha/2}$ can be taken from [9].

For $n_1, n_2 > 10$, the normal approximation

$$L^* = (L - E[L]) / \sqrt{D[L]} \tag{13}$$

is valid, where

$$E[L] = \frac{n_1}{n_1 + n_2} \sum_{i=1}^{n_1+n_2} u^2_{i/(n_1+n_2+1)};$$

TABLE 3. Power of Tests with Respect to the Competing Hypotheses $H_3 : \sigma_2 = 1.5\sigma_1$

Test	α	Sample size n				
		10	20	40	60	100
Mood	0.1	0.255	0.425	0.688	0.841	0.964
	0.05	0.158	0.302	0.565	0.751	0.931
	0.01	0.045	0.121	0.319	0.518	0.802
Ansari–Bradley	0.1	0.242	0.393	0.608	0.768	0.926
	0.05	0.150	0.270	0.484	0.659	0.869
	0.01	0.041	0.104	0.254	0.413	0.693
Siegel–Tukey	0.1	0.246	0.383	0.609	0.768	0.926
	0.05	0.155	0.261	0.484	0.659	0.869
	0.01	0.043	0.056	0.251	0.414	0.693

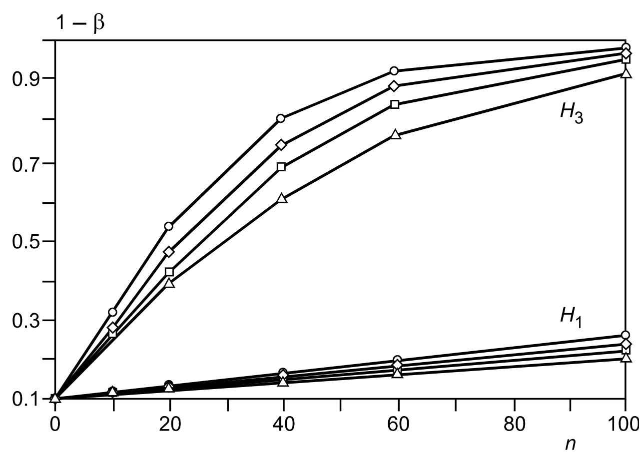

Fig. 2. Powers of tests with respect to the competing hypotheses H_1 and H_3 as functions of sample size n for $\alpha = 0.1$ in the case of a normal law: ○) Bartlett; ◇) Levene; □) Mood; △) Ansari–Bradley.

$$D[L] = \frac{n_1 n_2}{(n_1 + n_2)(n_1 + n_2 - 1)} \sum_{i=1}^{n_1+n_2} u^4_{i/(n_1+n_2+1)} - \frac{n_2}{n_1(n_1 + n_2 - 1} \left[\frac{n_1}{n_1 + n_2} \sum_{i=1}^{m+n} u^2_{i/(n_1+n_2+1)} \right].$$

The hypothesis of equal scale parameters is not rejected for $N^*_{\alpha/2} < L^* < N^*_{1-\alpha/2}$. The distributions of the (12) and (13) statistics are also quite smooth.

Comparative Analysis of Powers. Tables 1–3 list the powers of the Mood, Ansari–Bradley, and Siegel–Tukey tests with respect to the different competing hypotheses for normal-law samples obtained here for different levels of significance.

The analysis shows that the Mood test has a significant advantage in power, while the Ansari–Bradley and Siegel–Tukey tests are essentially equivalent. There is some “disparity” in these tables for sample sizes of $n = 10$ and 20; this is explained by the different degree of discreteness in the distributions of the statistics for these tests.

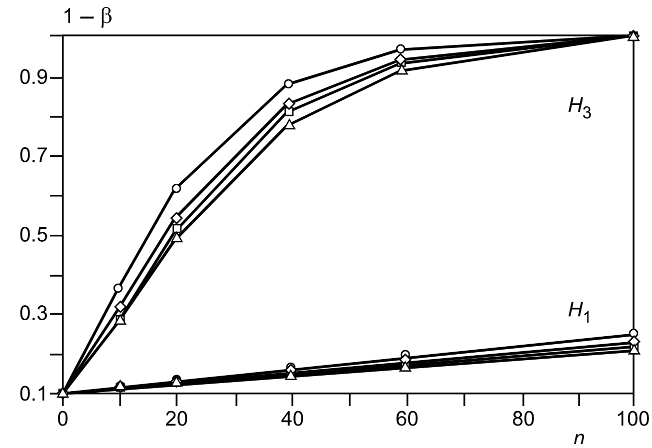

Fig. 3. Powers of multisample tests with respect to the competing hypotheses H_1 and H_3 as functions of the sample size n for $\alpha = 0.1$ in the case of a normal law for $m = 5$ analyzed samples; ○) Cochran; ◇) Bartlett; □) Hartley; △) Levene.

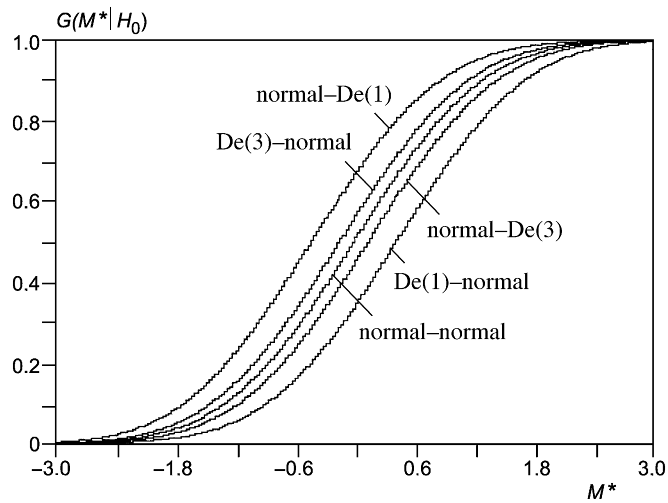

Fig. 4. Distributions of the normalized statistic for the Mood test when hypothesis H_0 is true when the pair of samples follow different laws from the family of Eq. (3).

The Capon and Klotz tests are extremely hard to use for samples of arbitrary size because of the need to use appropriate markers, which are only available for small sample sizes. A study of the power of these tests for sample sizes of up to 10 has shown that in this case their powers are no higher than that of the Mood test. Thus, there was no need for further study.

The power of the tests studied for different, non-normal distributions. We note that, as in the case of the parametric distributions [1], the power, with respect to the same competing hypotheses, of all the tests increases with “lightening” of the tails of the distributions (compared to normal) obeyed by the analyzed samples.

It is natural that the nonparametric tests are inferior in terms of power to the parametric distributions [1] of Bartlett, Cochran, Hartley, and Fisher. Figure 2 shows plots of the powers of the tests with respect to the competing hypotheses H_1 and H_3 as functions of the sample size n for $\alpha = 0.1$ in the case of a normal law. The considerable advantage of the Bartlett test compared to the Mood test, the most powerful of the nonparametric tests, can be seen here. Recall that the powers of the parametric Bartlett, Cochran, Hartley, and Fisher tests are the same when there are two samples [1]. However, for more than

TABLE 4. Upper Percentage Points for the Statistic of the Cochran Test in the Case of Two Samples of Equal Sizes n Belonging to Family (3) with Different Values of the Form Parameter θ_0 and $\alpha = 0.1, 0.05$, and 0.01

n	$\theta_0 = 1$			$\theta_0 = 2$			$\theta_0 = 3$			$\theta_0 = 4$			$\theta_0 = 5$		
	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
5	0.917	0.947	0.980	0.865	0.906	0.959	0.845	0.890	0.950	0.836	0.883	0.947	0.831	0.879	0.945
8	0.862	0.900	0.949	0.791	0.833	0.899	0.764	0.807	0.877	0.751	0.794	0.866	0.744	0.787	0.861
10	0.836	0.875	0.930	0.761	0.801	0.868	0.733	0.773	0.842	0.720	0.759	0.829	0.713	0.751	0.822
15	0.789	0.829	0.890	0.713	0.748	0.811	0.686	0.719	0.780	0.674	0.706	0.765	0.667	0.698	0.757
20	0.759	0.797	0.858	0.684	0.716	0.774	0.660	0.689	0.743	0.648	0.676	0.728	0.642	0.669	0.720
25	0.736	0.772	0.834	0.665	0.694	0.748	0.642	0.668	0.717	0.632	0.656	0.703	0.626	0.649	0.695
30	0.718	0.753	0.814	0.650	0.677	0.727	0.629	0.653	0.699	0.619	0.642	0.685	0.614	0.635	0.677
40	0.693	0.725	0.782	0.630	0.654	0.699	0.611	0.632	0.672	0.603	0.622	0.660	0.598	0.616	0.653
50	0.674	0.704	0.758	0.617	0.638	0.679	0.599	0.618	0.654	0.591	0.609	0.642	0.587	0.604	0.636
60	0.660	0.689	0.740	0.606	0.626	0.664	0.591	0.608	0.640	0.583	0.599	0.630	0.579	0.594	0.624
70	0.649	0.676	0.724	0.598	0.617	0.652	0.584	0.599	0.630	0.577	0.591	0.620	0.573	0.587	0.614
80	0.640	0.665	0.712	0.592	0.609	0.642	0.578	0.593	0.621	0.572	0.585	0.612	0.568	0.581	0.607
90	0.632	0.657	0.701	0.587	0.603	0.634	0.573	0.587	0.614	0.567	0.580	0.605	0.564	0.576	0.600
100	0.626	0.649	0.692	0.582	0.598	0.628	0.570	0.583	0.609	0.564	0.576	0.600	0.561	0.572	0.595

TABLE 5. Upper Percentage Points for the Statistic of the Cochran Test in the Case of Three Samples of Equal Sizes n Belonging to Family (3) with Different Values of the Form Parameter θ_0 and $\alpha = 0.1, 0.05$, and 0.01

n	$\theta_0 = 1$			$\theta_0 = 2$			$\theta_0 = 3$			$\theta_0 = 4$			$\theta_0 = 5$		
	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
5	0.794	0.847	0.918	0.700	0.752	0.839	0.665	0.717	0.806	0.649	0.700	0.790	0.641	0.690	0.781
8	0.716	0.768	0.852	0.614	0.658	0.741	0.579	0.620	0.698	0.563	0.602	0.677	0.554	0.591	0.665
10	0.681	0.732	0.817	0.581	0.622	0.698	0.548	0.584	0.654	0.533	0.567	0.634	0.524	0.557	0.622
15	0.623	0.669	0.751	0.531	0.564	0.628	0.503	0.531	0.588	0.489	0.516	0.569	0.482	0.508	0.558
20	0.587	0.629	0.707	0.502	0.531	0.588	0.477	0.501	0.550	0.466	0.488	0.533	0.459	0.480	0.524
25	0.562	0.600	0.673	0.484	0.509	0.560	0.461	0.482	0.526	0.450	0.470	0.510	0.444	0.463	0.501
30	0.543	0.578	0.647	0.470	0.493	0.539	0.449	0.468	0.507	0.439	0.457	0.493	0.434	0.451	0.485
40	0.515	0.547	0.608	0.450	0.470	0.510	0.432	0.449	0.482	0.424	0.439	0.470	0.419	0.434	0.463
50	0.496	0.525	0.581	0.437	0.455	0.490	0.421	0.436	0.465	0.414	0.427	0.454	0.410	0.422	0.448
60	0.482	0.508	0.560	0.428	0.444	0.476	0.413	0.426	0.453	0.406	0.418	0.443	0.402	0.414	0.437
70	0.471	0.495	0.543	0.421	0.435	0.465	0.407	0.419	0.444	0.401	0.412	0.434	0.397	0.408	0.429
80	0.462	0.485	0.530	0.415	0.429	0.456	0.402	0.413	0.436	0.396	0.406	0.427	0.393	0.403	0.422
90	0.455	0.476	0.518	0.410	0.423	0.449	0.398	0.408	0.430	0.392	0.402	0.422	0.389	0.398	0.417
100	0.449	0.469	0.509	0.406	0.418	0.443	0.394	0.405	0.425	0.389	0.398	0.417	0.386	0.395	0.413

TABLE 6. Upper Percentage Points for the Statistic of the Cochran Test in the Case of Four Samples of Equal Sizes n Belonging to Family (3) with Different Values of the Form Parameter θ_0 and $\alpha = 0.1, 0.05$, and 0.01

n	$\theta_0 = 1$			$\theta_0 = 2$			$\theta_0 = 3$			$\theta_0 = 4$			$\theta_0 = 5$		
	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
5	0.696	0.755	0.848	0.584	0.634	0.727	0.545	0.591	0.679	0.527	0.571	0.656	0.517	0.560	0.643
8	0.611	0.666	0.761	0.501	0.541	0.619	0.466	0.500	0.569	0.450	0.482	0.546	0.441	0.471	0.533
10	0.575	0.626	0.720	0.470	0.506	0.576	0.438	0.468	0.529	0.423	0.451	0.507	0.415	0.441	0.495
15	0.517	0.561	0.646	0.424	0.453	0.510	0.397	0.421	0.468	0.385	0.406	0.450	0.378	0.398	0.439
20	0.482	0.521	0.598	0.399	0.422	0.471	0.375	0.395	0.435	0.364	0.382	0.419	0.358	0.375	0.410
25	0.457	0.493	0.563	0.382	0.403	0.445	0.360	0.378	0.413	0.351	0.366	0.398	0.346	0.360	0.390
30	0.439	0.471	0.536	0.369	0.388	0.427	0.350	0.365	0.397	0.341	0.355	0.384	0.336	0.349	0.377
40	0.413	0.441	0.498	0.352	0.368	0.401	0.335	0.348	0.348	0.328	0.340	0.364	0.324	0.335	0.358
50	0.395	0.420	0.470	0.340	0.355	0.384	0.326	0.337	0.361	0.319	0.329	0.351	0.315	0.325	0.345
60	0.382	0.404	0.451	0.332	0.345	0.371	0.319	0.329	0.350	0.313	0.322	0.341	0.309	0.318	0.336
70	0.372	0.392	0.435	0.326	0.337	0.361	0.313	0.323	0.342	0.308	0.316	0.334	0.305	0.313	0.329
80	0.364	0.383	0.422	0.320	0.331	0.354	0.309	0.318	0.336	0.304	0.312	0.328	0.301	0.309	0.324
90	0.357	0.375	0.412	0.316	0.326	0.348	0.305	0.314	0.331	0.300	0.308	0.324	0.298	0.305	0.320
100	0.352	0.368	0.403	0.313	0.322	0.342	0.302	0.310	0.327	0.298	0.305	0.320	0.295	0.302	0.316

TABLE 7. Upper Percentage Points for the Statistic of the Cochran Test in the Case of Five Samples of Equal Sizes n and Belonging to Family (3) with Different Values of the Form Parameter θ_0 and $\alpha = 0.1, 0.05$, and 0.01

n	$\theta_0 = 1$			$\theta_0 = 2$			$\theta_0 = 3$			$\theta_0 = 4$			$\theta_0 = 5$		
	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01	0.1	0.05	0.01
5	0.623	0.684	0.787	0.504	0.551	0.642	0.464	0.505	0.588	0.446	0.484	0.562	0.436	0.472	0.548
8	0.537	0.591	0.690	0.426	0.461	0.533	0.392	0.421	0.482	0.376	0.403	0.458	0.367	0.393	0.446
10	0.501	0.550	0.645	0.397	0.428	0.491	0.366	0.392	0.444	0.352	0.375	0.422	0.344	0.366	0.411
15	0.445	0.485	0.567	0.355	0.379	0.429	0.330	0.349	0.390	0.318	0.336	0.372	0.312	0.329	0.363
20	0.412	0.447	0.520	0.332	0.352	0.394	0.310	0.326	0.360	0.300	0.315	0.345	0.295	0.308	0.337
25	0.388	0.420	0.485	0.316	0.334	0.371	0.297	0.311	0.341	0.288	0.301	0.328	0.283	0.295	0.320
30	0.370	0.399	0.459	0.305	0.321	0.354	0.287	0.300	0.327	0.279	0.291	0.315	0.275	0.286	0.308
40	0.347	0.371	0.371	0.290	0.303	0.331	0.275	0.285	0.308	0.268	0.278	0.298	0.264	0.273	0.292
50	0.330	0.352	0.397	0.280	0.291	0.316	0.266	0.276	0.295	0.260	0.269	0.286	0.257	0.265	0.281
60	0.318	0.337	0.378	0.272	0.283	0.304	0.220	0.227	0.242	0.254	0.262	0.278	0.252	0.259	0.274
70	0.309	0.326	0.363	0.266	0.276	0.296	0.255	0.263	0.279	0.250	0.257	0.272	0.247	0.254	0.268
80	0.301	0.318	0.352	0.262	0.271	0.289	0.251	0.259	0.274	0.247	0.253	0.267	0.244	0.250	0.263
90	0.295	0.310	0.342	0.258	0.266	0.284	0.248	0.255	0.269	0.244	0.250	0.263	0.242	0.247	0.259
100	0.290	0.304	0.334	0.255	0.263	0.279	0.246	0.252	0.265	0.242	0.247	0.259	0.239	0.245	0.256

two samples, when use of the nonparametric criteria is not provided for, the difference in power of the Bartlett, Cochran, and Hartley tests is perceptible.

Figure 3 shows graphs of the powers of multisample tests with respect to the same competing hypotheses H_1 and H_3 as functions of the sample size n for $\alpha = 0.1$ in the case of a normal law for $m = 5$ samples to be analyzed.

During discussions of the advantages and disadvantages of parametric and nonparametric tests, radical opinions are sometimes advanced to the effect that only nonparametric tests should be used. This is justified by the argument that in practice the distribution for the analyzed samples is not known and, usually, differs from normal. In that case, it is not possible to use parametric tests of the homogeneity of variances using the classical results based on the assumption of a normal distribution. Nonparametric tests do not require that this condition be met. This is indeed so. If both samples adhere to one and the same general set, the distributions $G(S \mid H_0)$ of the statistics for the nonparametric tests are independent of the form of the assumed law.

However, if when the hypothesis H_0 of equal dispersions is true, the samples obey different laws, then the distribution $G(S \mid H_0)$ is found to depend on these laws. As an example confirming this, Fig. 4 shows the distributions of the normalized statistic (7) for the Mood test when two samples of the same size $n = 10$ with equal dispersions follow different pairs of distributions from the (3) family. Figure 4 implies that the distribution also depends on the ordering of the distributions obeyed by the samples.

This feature is also characteristic of parametric tests, but in the analogous situation we believe that the changes in, for example, the distributions of the statistics for the Cochran test turn out to be somewhat smaller.

Use of the Cochran Test for Non-Normal Laws. In comparisons of the values of the powers of parametric (Tables 1–3 of [1]) and nonparametric (Tables 1–3) tests, there is a considerable advantage in the power of the parametric tests. This advantage even remains in situations where the analyzed samples are far from normal. In the latter case, it is impossible to use the classical results associated with the distributions (or percentage points) of the statistics for the Bartlett, Cochran, Hartley, and Levene tests. The problem is made more complicated by the fact that, when the classical assumptions of normality break down, the distributions of the statistics for these tests for a true test hypothesis depend on the distributions of the analyzed samples and on the sample sizes. In principle, we have the same thing, for example, for the Cochran, Hartley, and Levene tests with a normal law. Thus, constructing (finding) a model for the distribution of the test statistic for arbitrary distributions and sizes of the samples is an unreal problem. However, for specific parametric models of distributions which come recommended for various applications as models of observed random quantities, this problem (as for a normal law) is relatively easy to solve using computer techniques [12, 13].

The Cochran distribution, which is at least as good as any other in the case of two samples, was shown to be preferable in Ref. 1 and, in its multisample version, it turns out to be the most powerful (except for distributions with “heavy” tails, where the Levene test comes well recommended).

In situations where the observed quantities have distributions from the (3) family with shape parameters $\theta_0 = 1–5$, and various sample sizes n, statistical modelling has been used to construct Tables 4–7, which list the upper percentage points (1, 5, 10%) of the statistic for the Cochran test (statistic 5 of [1]) for two samples. The resulting percentage points can be used when there is reason to assume that the (3) distribution with the corresponding shape parameter θ_0 is a good model for the observed random quantities. These percentage points are an improvement on some results given in [14] and extend the prospects for using the Cochran test.

Conclusion. We now summarize this study of the use of tests of hypotheses regarding the homogeneity of dispersions and point out the following basic facts.

For analyses of two samples, the Fisher, Bartlett, Cochran, and Hartley tests have the same power with respect to the same competing hypotheses. When analyzing more than two samples for homogeneity, the Cochran test has the advantage in terms of power.

Of the nonparametric tests examined here, the Mood test has the most power, but it is significantly lower than the Fisher, Bartlett, Cochran, Hartley, and Levene tests in this regard.

When necessary, the effect of the parametric tests can be extended to situations where the samples are described by non-normal laws if computer modelling is used to study the distributions of the statistics and to construct models or tables of percentage points for these distributions. Of the tests examined here, the Cochran test is most suitable for this role. However, it should be noted that the distribution of its statistic will depend on the type of distribution law and volume of the samples and, in many cases, on the particular values of some parameters of those laws. Solving these problems by means of suitable programs [7] does not involve any fundamental difficulties and can be done as needed [15]. We note that the specific nature of the problems to be studied by computer modelling of probabilistic behavior allows efficient use of parallel computing, employing the resources of multicore and multiprocessor computers and computer networks, with solutions obtained on an almost real-time scale.

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