About properties and power of classical tests of homogeneity of variances

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***Abstract*—Distributions of test statistics of classical tests for homogeneity of variance (Neyman–Pearson, O'Brien, Link, Newman, Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, Overall–Woodward Z-variance and modified Overall–Woodward Z-variance tests) are investigated including a case when the standard assumption of the normality is violated. The comparative analysis of power of the classical tests is carried out. Method of application of the tests of violation of the standard assumption that provides an interactive simulation of distributions of the test statistics is proposed and tested.**

Keywords*—test of homogeneity of variances, Neyman–Pearson test, O'Brien test, Link test, Newman test, Bliss–Cochran–Tukey test, Cadwell–Leslie–Brown test, Overall–Woodward Z-variance test, modified Overall–Woodward Z-variance test, power of test

I. INTRODUCTION

Tests of homogeneity of variances are frequently used in various applications. The tasks of processing of measuring results are no exception. Perhaps, the most striking example of demand for tests of homogeneity of variances in the area of metrology is the task of comparison of laboratory tests.

The hypothesis of constant variances of m samples and the competing hypothesis have the form

$$H_0:\ \sigma_1^2=\sigma_2^2=\ldots=\sigma_m^2,\qquad H_1:\ \sigma_{i_1}^2\neq\sigma_{i_2}^2, \tag{1}$$

where the inequality holds for at least one pair of indices $i_1,\ i_2$. Some tests can be used only for $m=2$.

The quality of statistical conclusions that is carried out by the results of analysis is provided with correctly application of corresponding tests that have the best power.

The standard assumption to determine the possibility of application of classical tests of homogeneity of variances is that the samples follow a normal distribution. This condition sharply limits the area of application of classical tests. This restriction is not imposed on nonparametric tests to test the hypothesis of equality of scaling parameters. However, on nonparametric tests the samples should belong to the same type of distribution.

In this paper, the conclusions of [1-4] have been added to results of comparative analysis of number of classical tests for homogeneity of variances (Neyman–Pearson [6], O'Brien [7], Link [8], Newman [9], Bliss–Cochran–Tukey [10], Cadwell–Leslie–Brown [11], Overall–Woodward Z-variance [12] and modified Overall–Woodward Z-variance tests [13]). The purpose of the work is to study the distributions of the statistics of these tests, to extend the table of percentage points, to make the comparative analysis of the power of the tests, to realize the feasibility of using the tests when the standard assumption is violated.

A study of the distributions of the statistics and an estimate of the power of the tests with respect to various alternative hypothesis have been done using a method of statistical simulation in the framework of the Windows Controlled Interval Statistics (ISW) program system. The number of statistical experiments for simulation of samples of the statistics was $N=10^6$. Than the difference between the true distribution of the statistics and the simulated empirical distribution usually is less than 10^{-3} in absolute value.

When the standard assumption of normality is violated the distributions of the test statistics are studied in a case where the simulated samples belong to a family with the density

$$f(x)=\frac{\theta_0}{2\theta_1\Gamma(1/\theta_0)}\exp\left(-\left(\frac{|x-\theta_2|}{\theta_1}\right)^{\theta_0}\right) \tag{2}$$

for different values of the shape parameter θ_0. The distribution $De(\theta_0)$ includes the normal $\theta_0=2$ and the Laplace $\theta_0=1$ distribution as special cases.

II. THE NEYMAN-PEARSON TEST

The statistics of the test [6] is a ratio between arithmetic mean of the all estimate of variances s_i^2 and geometric mean:

$$h=\frac{1}{m}\sum_{i=1}^{m}s_i^2\Bigg/\left(\prod_{i=1}^{m}s_i^2\right)^{\frac{1}{m}}, \tag{3}$$

where m is the number of samples, $s_i^2 = \frac{1}{n_i - 1}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{x}_i\right)^2$ is estimate of sample variances, $\overline{x}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} x_{ij}$ is the mean of i-th sample, x_{ij} – j-th observation in the i-th sample. It is assumed that $n_1 = n_2 = \ldots = n_m = n$. The test is right-sided. The hypothesis H_0 is rejected when $h > h_{1-\alpha}$.

The distributions of test statistic (4) depend on n and m. In this work, the values of percentage points have been refined for distributions of statistic. The Neyman-Pearson test is very sensitive to any departures from normality (Fig. 1).

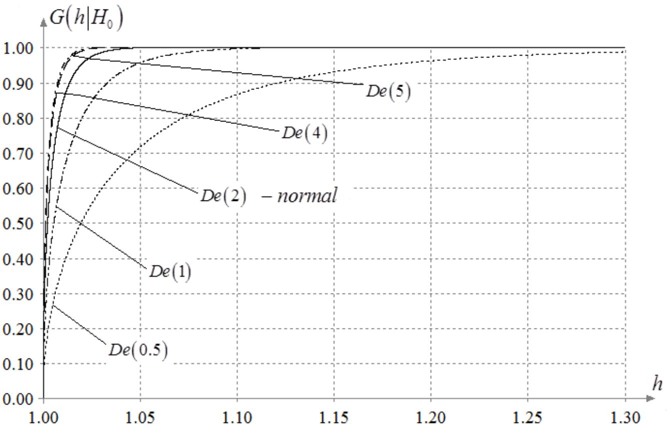

Fig. 1. Distributions of the statistic of the Neyman-Pearson test depending on the type of distributional law, for $n = 100$, $m = 2$.

Naturally, that the test can be used for unequal n_i. However, in this case, the distribution of the statistics for true null hypothesis H_0 differs from the distribution with equal n_i.

III. THE O'BRIEN TEST

Every raw score x_{ij} is transformed using the following formula [7]:

$$V_{ij} = \frac{(n_i - 1.5) n_i \left(x_{ij} - \overline{x}_i\right)^2 - 0.5 s_i^2 (n_i - 1)}{(n_i - 1)(n_i - 2)}, \qquad (4)$$

where n_i is sample size, $\overline{x}_i$ is the mean, s_i^2 – the unbiased estimate of variance for i-th sample.

The test statistic is:

$$V = \frac{\frac{1}{m-1}\sum_{i=1}^{m} n_i \left(\overline{V}_i - \overline{\overline{V}}_i\right)^2}{\frac{1}{N-m}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(V_{ij} - \overline{V}_i\right)^2}, \qquad (5)$$

where $\overline{V}_i = \frac{1}{n_i}\sum_{j=1}^{n_i} V_{ij}$, $\overline{\overline{V}}_i = \frac{1}{N}\sum_{i=1}^{m}\sum_{j=1}^{n_i} V_{ij}$, $N = \sum_{i=1}^{m} n_i$.

The test is right-sided. If the test statistic (5) exceeds the critical value the null hypothesis H_0 is rejected. When the null hypothesis is true the statistic of the O'Brien test has approximately $F_{m-1,N-m}$–distribution [7]. Nevertheless, the study has shown that the distribution of statistic (5) converges quite slowly to $F_{m-1,N-m}$-distribution. For example, in the case of $m = 2$, difference between real distribution $G\left(V|H_0\right)$ of statistic (5) and conforming $F_{1,N-m}$-distribution can be neglected only when $n_1 = n_2 = n \geq 80$. For small sample sizes, the essential difference is for large values V therefore using of percentage points for the $F_{m-1,N-m}$-distribution increases the probability of a Type 2 error in consequence of decrease of a set significance level α.

The upper critical values have been obtained for different number m of compared samples for $n_1 = n_2 = n \leq 80$ to provide the possibility of correctly using of the test for small sample sized.

For $N - m \leq 80$, the distributions $G\left(V|H_0\right)$ for values V such that $1 - G\left(V|H_0\right) < 0.1$ are nearer to $F_{m-1,\infty}$-distribution than the $F_{m-1,N-m}$-distribution. Therefore, in these situations, correctness of results can be increased using $F_{m-1,\infty}$-distribution to estimate the achieved significance level (p_{value}) or choosing critical values $V_{1-\alpha}$ according to $F_{m-1,\infty}$-distribution.

Distributions of statistic of the O'Brien test are quite robust to violation of the assumption of normality. If the tails of distribution are "easier" than tails of normal distribution, the test statistic doesn't change significantly. If the tails are "heavier", deviations are smaller than deviations for other classical tests. Only the modified Overall–Woodward Z-variance tests has the similar robustness to deviation from normality among the testы that are considered in this work.

IV. THE LINK TEST

The Link test is analogue of Fisher test using only for analysis of two ($m = 2$). The test statistic is defined as [8]:

$$F^* = \frac{\omega_{n_1}}{\omega_{n_2}}, \qquad (6)$$

where $\omega_{n_1} = x_{1,\max} - x_{1,\min}$, $\omega_{n_1} = x_{2,\max} - x_{2,\min}$ is ranges of samples.

The test is two-sided. The hypothesis is rejected if $F^* > F^*_{1-\alpha/2}$ or $F^* < F^*_{\alpha/2}$, where α is significance level, $F^*_{1-\alpha/2}$ and $F^*_{\alpha/2}$ is upper and lower critical values of statistic.

The distribution of test statistic depends essentially on sample sizes. The Link test is very sensitive to any violation of standard assumption. Upper and lower critical values of statistic (6) have been refined using the methods of statistical simulation.

V. THE NEWMAN TEST

The statistic of Newman test is defined as follows [9]:

$$q = \frac{\omega_{n_1}}{s_{n_2}}, \tag{7}$$

where $\omega_{n_1} = x_{1,\max} - x_{1,\min}$, $s_{n_2} = \sqrt{\frac{1}{n_2 - 1}\sum_{i=1}^{n_2}\left(x_{2i} - \overline{x}_2\right)^2}$.

As the previous test, the Newman test is two-sided. If the null hypothesis H_0 is true, the distribution of test statistics (7) depends on sample sizes and the law of distribution of samples. In this work, upper and lower critical values of statistic (7) have been refined.

VI. THE BLISS–COCHRAN–TUKEY TEST

The test [10] was proposed as analogue of the Cochran test

$$c = \frac{\max\limits_{1 \le i \le m} \omega_i}{\sum\limits_{i=1}^{m} \omega_i}, \tag{8}$$

where m is number of samples, $\omega_i = \max\limits_{1 \le j \le n_i} x_{ij} - \min\limits_{1 \le j \le n_i} x_{ij}$ is range of i-th sample.

The test is right-sided. The distribution of test statistic highly depends on sample sizes. As and the Cochran test, the distributions of the test statistic are very sensitive to departure from normality.

VII. THE CADWELL–LESLIE–BROWN TEST

The test [11] is analogue the Hartley test:

$$K = \frac{\max\limits_{1 \le i \le m} \omega_i}{\min\limits_{1 \le i \le m} \omega_i}, \tag{9}$$

where m is sample sizes, ω_i is range of i-th sample.

The test is right-sided. The distribution of statistic of the Cadwell–Leslie–Brown test, as the distribution of Bliss-Cochran test-statistic, depends essentially on sample sizes and on law of distribution.

In this work, critical values $K_{1-\alpha}$ have been refined for different number of samples m and equal sample sizes $n_i = n,\ i = \overline{1,m}$. The study shows, that most of the tests significantly exceed the Cadwell–Leslie–Brown test on the power when the sample sizes are large.

VIII. THE OVERALL–WOODWARD Z-VARIANCE TEST

The test statistic is written as [12]:

$$Z = \frac{1}{m-1}\sum_{i=1}^{m} Z_i^2, \tag{10}$$

where m is number of samples, $Z_i = \sqrt{\frac{c_i\left(n_i - 1\right)s_i^2}{MSE}} - \sqrt{c_i\left(n_i - 1\right) - \frac{c_i}{2}}$, $c_i = 2 + 1/n_i$, $MSE = \frac{1}{N-m}\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(x_{ij} - \overline{x}_i\right)^2$, n_i is size of i-th sample, s_i^2 – is the unbiased estimate of sample variances, $N = \sum_{i=1}^{m} n_i$.

If the null hypothesis H_0 is true and the samples obey normal law of distribution, the distribution of test statistic (10) has approximately $F_{m-1,\infty}$-distribution and doesn't depend on sample sizes. However, for small sample sizes, distribution of statistic differs significantly from $F_{m-1,\infty}$-distribution. The analysis has shown, that difference between real and $F_{m-1,\infty}$-distribution can be neglected, if sample sizes $n \ge 50$. In case when assumption of normality was satisfied and sample sizes are $n \le 50$, upper critical values $Z_{1-\alpha}$ have been computed using methods of statistical simulation.

As with most of the classical tests for homogeneity of variance, distribution of Z-variance test statistic is very sensitive to violation of assumption of normality.

IX. THE MODIFIED OVERALL–WOODWARD Z-VARIANCE TEST

Overall and Woodward proposed modification of Z-variance test [13] to construct test that would remain stable when sample data deviate from normality. The new values c_i depends on the sample sizes and the mean of kurtosis indices:

$$c_i = 2.0\left[\frac{1}{K_i}\left(2.9 + \frac{0.2}{n_i}\right)\right]^{\frac{1.6\left(n_i - 1.8K_i + 14.7\right)}{n_i}}, \tag{11}$$

where $K_i = \frac{1}{n_i - 2}\sum_{j=1}^{n_i} G_{ij}^4$ – estimate of kurtosis index of i-th sample, $G_{ij} = \left(x_{ij} - \overline{x}_i\right) \Big/ \sqrt{\frac{n_i - 1}{n_i} s_i^2}$, $\overline{K}$ – the mean of kurtosis indices.

The studies were demonstrated, that the distribution of test statistic converges slowly to $F_{m-1,\infty}$-distribution with increased sample sizes. Even when the sample sizes are large values, the distribution of test statistic differs from $F_{m-1,\infty}$- distribution. However, in the range of large values of the statistic, difference between distribution of test statistic and $F_{m-1,\infty}$-distribution is not significant. Critical values were found to apply the test correctly for small sample sizes.

At the same time, it should be noted, that the distribution of statistic of the modified Overall-Woodward Z-variance test really is more robust to departures from normality. The obvious difference between distribution of modified test statistic for symmetric laws of distributions and distribution of modified test statistic for normal law of distribution is only for heavy tails. The struggle for robustness led to a decrease of power.

X. THE COMPARITIVE ANALISYS OF POWER OF THE TESTS

The analysis of power of the tests was made concerning the alternative hypotheses ($H_1 : \sigma_m = 1.1\sigma_0$, $H_2 : \sigma_m = 1.2\sigma_0$, $H_3 : \sigma_m = 1.5\sigma_0$). The estimates of power of classical Bartlett, Cochran, Levene, Hartley, Fisher tests [2] and nonparametric Mood, Ansari-Bradley, Siegel-Tukey tests [3] were included in the comparative analysis.

The obtained estimates of power of the tests for the case of normal samples for significance levels $\alpha = 0.1,\ 0.05,\ 0.01$ and number of samples $m = 2$ are shown in descending order of power in Tables 1-2.

TABLE I. POWER OF TESTS OF HOMOGENEITY OF VARIANCES RELATIVE TO ALTERNATIVE HYPOTHESIS $H_1 :\ \sigma_2 = 1.1\sigma_1$

Test	α	**Sample sizes**				
		$n=10$	$n=20$	$n=40$	$n=60$	$n=100$
Bartlett, Cochran, Hartley, Fisher, Neyman–Pearson, Z-variance	0.1	0.112	0.128	0.157	0.188	0.246
	0.05	0.058	0.068	0.090	0.111	0.156
	0.01	0.012	0.016	0.023	0.032	0.051
O'Brien	0.1	0.109	0.125	0.154	0.184	0.243
	0.05	0.056	0.066	0.087	0.108	0.153
	0.01	0.012	0.015	0.022	0.030	0.049
Modified Z-variance	0.1	0.110	0.123	0.150	0.176	0.228
	0.05	0.056	0.065	0.084	0.103	0.141
	0.01	0.012	0.014	0.021	0.028	0.044
Levene	0.1	0.111	0.123	0.143	0.159	0.186
	0.05	0.057	0.066	0.080	0.091	0.112
	0.01	0.012	0.015	0.020	0.025	0.033
Newman	0.1	0.112	0.128	0.157	0.188	0.246
	0.05	0.058	0.068	0.090	0.111	0.156
	0.01	0.012	0.016	0.023	0.032	0.051
Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, Link	0.1	0.111	0.119	0.133	0.141	0.154
	0.05	0.057	0.063	0.072	0.078	0.087
	0.01	0.012	0.014	0.018	0.019	0.023
Mood	0.1	0.111	0.120	0.143	0.166	0.211
	0.05	0.057	0.064	0.080	0.096	0.128
	0.01	0.012	0.014	0.020	0.026	0.039
Ansari-Bradley	0.1	0.101	0.125	0.135	0.154	0.190
	0.05	0.052	0.064	0.074	0.087	0.113
	0.01	0.011	0.014	0.019	0.023	0.033
Siegel-Tukey	0.1	0.106	0.121	0.135	0.154	0.190
	0.05	0.055	0.062	0.075	0.087	0.113
	0.01	0.011	0.010	0.018	0.023	0.033

TABLE II. POWER OF TESTS OF HOMOGENEITY OF VARIANCES RELATIVE TO ALTERNATIVE HYPOTHESIS $H_3 :\ \sigma_2 = 1.5\sigma_1$

Test	α	**Sample sizes**				
		$n=10$	$n=20$	$n=40$	$n=60$	$n=100$
Bartlett, Cochran, Hartley, Fisher, Neyman–Pearson, Z-variance	0.1	0.312	0.532	0.806	0.926	0.991
	0.05	0.201	0.402	0.705	0.871	0.980
	0.01	0.064	0.182	0.463	0.692	0.924
O'Brien	0.1	0.266	0.490	0.783	0.917	0.990
	0.05	0.155	0.344	0.664	0.849	0.976
	0.01	0.039	0.127	0.379	0.628	0.903
Modified Z-variance	0.1	0.265	0.489	0.781	0.916	0.990
	0.05	0.158	0.348	0.666	0.849	0.976
	0.01	0.043	0.138	0.397	0.639	0.906
Levene	0.1	0.269	0.471	0.746	0.888	0.981
	0.05	0.163	0.338	0.628	0.812	0.960
	0.01	0.045	0.131	0.364	0.590	0.866
Newman	0.1	0.296	0.473	0.682	0.796	0.901
	0.05	0.190	0.348	0.566	0.699	0.840
	0.01	0.060	0.153	0.326	0.473	0.667
Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, Link	0.1	0.285	0.425	0.584	0.674	0.776
	0.05	0.181	0.305	0.458	0.554	0.671
	0.01	0.057	0.127	0.237	0.314	0.430
Mood	0.1	0.255	0.425	0.688	0.841	0.964
	0.05	0.158	0.302	0.565	0.751	0.931
	0.01	0.045	0.121	0.319	0.518	0.802
Ansari-Bradley	0.1	0.242	0.393	0.608	0.768	0.926
	0.05	0.150	0.270	0.484	0.659	0.869
	0.01	0.041	0.104	0.254	0.413	0.693
Siegel-Tukey	0.1	0.246	0.383	0.609	0.768	0.926
	0.05	0.155	0.261	0.484	0.659	0.869
	0.01	0.043	0.056	0.251	0.414	0.693

The Neyman-Pearson, Overall-Woodward Z-variance Bartlett, Cochran, Hartley and Fisher tests appear to be equivalent in power. Difference between modified Overall-Woodward Z-variance and O'Brien tests is noticeable only if the competing hypothesis is relatively distant (H_3). At the same time, both tests have an advantage in power over the Levene test. Note that the O'Brien, Levene and modified Z-variance tests are relatively robust to violation of normal assumption.

The Newman test is inferior to the Levene test in power with an increase in sample sizes. At the same time, the Newman test is superior to Bliss-Cochran-Tukey, Cadwell-Lesley-Brown and Link in power (except when $n = 10$). The last three tests are equivalent in power.

It should be noted, that for small sample sizes ($n = 10$), group of robust tests (modified Z-variance, O'Brien and Levene tests) is inferior to the Newman, Link, Bliss-Cochran-Tukey, Cadwell-Lesley-Brown tests in power, but with the increase of sample sizes n, has a significant advantage over these tests and nonparametric tests. In the robust group the O'Brien test has a slight advantage.

The Newman, Bliss-Cochran-Tukey, Cadwell-Lesley-Brown are more powerful than nonparametric tests only when sample sizes are small ($n = 10 \div 20$).

The Bartlett, Cochran, Hartley, Levene, Neyman-Pearson, O'Brien, Bliss-Cochran-Tukey, Cadwell-Lesley-Brown, Overall-Woodward Z-variance and modified Z-variance tests can be used when the number of samples is more than two. However, The Bartlett, Cochran, Hartley, Neyman-Pearson and Overall-Woodward Z-variance test are not equivalent in power anymore.

The table 3 contains the obtained estimates of power of the multisample tests for the case of normal samples for significance levels $\alpha = 0.1$, 0.05, 0.01 and number of samples $m = 3$ and $m = 5$ relative to alternative hypothesis H_3.

TABLE III. POWER OF MULTISAMPLE TESTS OF HOMOGENEITY OF VARIANCES RELATIVE TO ALTERNATIVE HYPOTHESIS $H_3:\ \sigma_m = 1.5\sigma_1$, $n_i = 100$, $i = \overline{1,m}$

Test	α					
	0.1	0.05	0.01	0.1	0.05	0.01
	$m=3$			$m=5$		
Cochran	0.997	0.994	0.974	0.998	0.997	0.987
O'Brien	0.996	0.990	0.961	0.997	0.994	0.976
Z–variance	0.996	0.991	0.964	0.997	0.993	0.974
Neyman-Pearson, Bartlett	0.996	0.990	0.962	0.996	0.992	0.970
Modified Z-variance	0.995	0.989	0.955	0.996	0.991	0.967
Hartley	0.995	0.988	0.947	0.995	0.989	0.955
Levene	0.990	0.979	0.926	0.991	0.982	0.944
Bliss–Cochran–Tukey	0.820	0.728	0.501	0.829	0.742	0.524
Cadwell–Leslie–Brown	0.795	0.691	0.444	0.783	0.675	0.432

The Cochran test performed the best in power with clear advantage. The next best is the O'Brien test. However when the number of samples is three and the hypothesis is close competing, the test hasn't advantages over the Overall-Woodward Z-variance, Neyman-Pearson and Bartlett tests. At the same time, the O'Brien test is more powerful than modified Z-variance and Levene tests that are robust to violation of the standard assumption of normality.

XI. CONCLUSION

As follows from the studies, correctness of statistical conclusions, that is carried out at test of hypothesis using classical tests for homogeneity of variances, directly depends on knowledge of the law of distribution of statistic for true null hypothesis H_0. Frequently, even when the assumption of normal distributions holds, distribution of test statistic is unknown and differs from asymptotic distribution. As a result, p-value can not be estimated.

When the standard assumption of normality is violated and the samples have some other type of distribution, statistics of the tests, as a rule, are unknown. Consequently, conclusion of the results of testing hypothesis can not be drawn.

However, if assumptions about estimated type of law of distribution can be justified, problem is not unsolvable. Based on the methods of statistical simulation (and corresponding software), the distributions of test statistics can be found in the process of the analysis, as in [14-15], including the interactive computing [15].

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