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COMPUTATIONAL AND DATA ACQUISITION SYSTEMS

Effect of the Roundoff on the Properties of Criteria for Testing Statistical Hypotheses

B. Yu. Lemeshko[1]* and S. B. Lemeshko[1]

[1] *Novosibirsk State Technical University, Novosibirsk, 630073 Russia*

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Abstract—The results of numerical studies of the effect of roundoff errors on the distribution of test statistics of statistical hypotheses are presented. The effect of the roundoff on the distribution of statistics of different goodness-of-fit and homogeneity tests is investigated. It is shown that, when the roundoff errors in the analyzed samples are comparable with the standard deviation of the measurement errors, the distribution of test statistics may vary significantly. Under such conditions the application of a test in the systems of data processing using the classical results may lead to false conclusions. The recommendations for resolving this issue are proposed.

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1. INTRODUCTION

In the analysis of measurement results coming from different sources, statistical methods are often used to decide whether the previously observed laws in the systems of data processing remain unchanged. For these purposes various criteria for testing statistical hypotheses may be applied.

Any measurements are accompanied by the roundoff error depending on the resolution of measurement system, including the characteristics of used sensors and analog-to-digital transducers. It is clear that the presence of roundoff has an impact on the results of application of statistical methods, and the effect of such errors may in some situations lead to false statistical conclusions.

It was noted in [1] that there may occur problems in application of normality tests and these problems are consequence of roundoff. The effect of roundoff errors on the real significance value was shown in [2, 3] with the use of example tests of hypotheses about the equality of mathematical expectation and variance to the nominal values and about Student's t-test on homogeneity of mean values and Fisher's criteria about the homogeneity of variances of two samples; in addition, it was pointed out that, as they increase, the power of tests decreases. It was noted in [4] in the analysis of a set of samples with repeating observations that in this situation the critical values of the distributions of statistics of non-parametric goodness-of-fit tests in checking composite hypotheses relative to the generalized Pareto distribution are different from those presented in [5]. However, in these works it was not discussed how the distributions of test statistics change as the roundoff errors increase.

The majority of existing tests are intended to test statistical hypotheses with respect to continuous random values. This assumption seldom is a focus of attention, but it determines correctness of application of the corresponding tests. When this assumption is satisfied, there cannot be repeated values in samples. In real situations, due to roundoff errors this assumption is often violated, which is typical for medical and biological experiments, where, because of their special character, the roundoff errors may be particularly large. This also concerns the results of high-precision measurements, in which only the last decimal sign is varied, which is related with the resolution of the used measurement

*E-mail: Lemeshko@ami.nstu.ru

system. In automated systems of data processing, one usually deals with rounded measurement results received from different sensors.

Let us explain what are the consequences of application, for instance, of the goodness-of-fit test, where the statistic takes into account the deviation of the empirical distribution from the theoretical one if the measurement results are rounded off with some Δ.

Suppose, to check a simple hypothesis H_0: $F_n(x) = F(x)$, where $F_n(x)$ is the empirical distribution constructed by the sample $x_1, x_2, \ldots, x_n$ of size n, we apply the goodness-of-fit test with statistic S. Suppose that there exists a limit distribution $G(S|H_0)$ of the statistic for this test. When H_0 is true, the empirical distribution $F_n(x)$, corresponding to the sample of continuous random values (without roundoff), converges to the distribution function $F(x)$ of this random value for $n \to \infty$. The empirical distribution $G_N(S_n|H_0)$ of statistic, constructed by the samples, will converge to the limit distribution of this statistic for $n \to \infty$ and for the number of experiments $N \to \infty$.

If the assumption of continuity of the observed random variable is violated and the measurement results are rounded off with some Δ, then, starting from some n (dependent on the form of $F(x)$, on the domain of definition of random value, and on Δ), $\max|F_n(x) - F(x)|$ stops decreasing, and the distribution $G_N(S_n|H_0)$ will deviate from the limit one $G(S|H_0)$ as n increases (the larger is Δ, the lower n is needed to make this deviation significant).

It was studied in [6] in testing simple and composite hypotheses how the distributions of statistics of the goodness-of-fit tests of Kolmogorov, Cramér–Mises–Smirnov, and Anderson deviate from the corresponding limit distributions in dependence on Δ as n increases. The picture similar to [6] (in the case of large samples) is also observed for a wide variety of criteria. To use the classical results in application of statistical tests under the conditions of large samples, it is recommended in [6] to restrict the amounts of the extracted samples by the value $n_{\max}$, at which the real distribution of statistic $G(S_{n_{\max}}|H_0)$ is still practically identical to $G(S|H_0)$. The assessment of the value $n_{\max}$ for the applied test in a specific situation is not problematic.

In many applications it is typical when relatively many repeated observations appear in the analyzed samples due to roundoff. It is explained by the fact that the roundoff error is comparable with the standard deviation of the distribution law of the measurement error. In these situations the real distributions $G(S_n|H_0)$ of test statistics (at the roundoff error Δ) may be significantly different from the limit distributions $G(S|H_0)$ or from $G(S_n|H_0)$, which occur in the case without rounding off the measurements.

In [7–10] the results of studying real properties of various groups of tests are presented without account for the effect of the roundoff errors on these properties. In the current case, exemplified by different tests, using the methods of statistical modeling, we show (i) how the roundoff error may influence the distribution of test statistics for testing various hypotheses at relatively small sizes of samples and (ii) what must be done to provide correctness of statistical conclusions in application of tests in these conditions. To conduct the investigations, it is implemented in the computational system [11], in which the list of tests slightly exceeding the number of tests covered in [7–10], is represented, that it is possible to apply this list and model the distributions of statistics of corresponding tests under conditions of violation of the standard assumption about continuity (at a given roundoff error Δ).

The proposed work is aimed at two goals. Firstly, we aim to show that the presence of roundoff errors often lead to the situations where the use of classical results concerning the criteria for testing statistical hypotheses appears to be absolutely impossible. Secondly, we aim to demonstrate the possibility of correct application of tests also in such situations.

For determinacy (and without loss in generality), in the performed studies we rely on the sets modeled according to the standard normal law $N(0,1)$, but for different roundoff errors. At the roundoff with $\Delta = 1$, in the samples belonging to $N(0,1)$ there may occur 9 unique values, and at the roundoff with $\Delta = 0.1$ there may occur approximately 86 unique values, at $\Delta = 0.01$ there are approximately 956 values, and at $\Delta = 0.001$ there are approximately 9830. In the case when the sample belong to the law $N(\mu,\sigma)$, the dependences demonstrated below have the same form at the roundoff error $\Delta\sigma$. The picture does not change qualitatively for the laws different from the normal one.

2. EFFECT OF THE ROUNDOFF ON THE VARIATION IN PROPERTIES OF GOODNESS-OF-FIT TEST

The statistic of Pearson's χ^2 test has the form

$$X_n^2 = n \sum_{i=1}^{k} \frac{(n_i/n - P_i(\theta))^2}{P_i(\theta)}, \tag{1}$$

where n_i is the number of observations falling into the ith interval, $P_i(\theta) = \int\limits_{x_{i-1}}^{x_i} f(x,\theta)dx$ are the probabilities of falling into the interval that correspond to the theoretical law with the density function $f(x,\theta)$. When the simple checked hypothesis H_0 is true (at known θ), χ^2_{k-1}-distribution is the asymptotic distribution of statistic.

In Fig. 1 we show how the distribution of statistic (1) of Pearson's test varies depending on the degree of rounding off Δ in testing simple hypothesis about the goodness-of-fit with the normal law at $n = 100$ and at the number of equally probable intervals $k = 10$.

At such amount of samples and at $\Delta = 0.01\sigma$, the distribution of statistic X_n^2 is practically identical to χ^2_9-distribution, but already at $\Delta = 0.05\sigma$ the difference becomes significant. As n increases, the picture presented in the figure extends from the χ^2_9-distribution and shrinks to the χ^2_9-distribution with decreasing n.

The variations in distributions of statistics of non-parametric goodness-of-fit tests [12] of Kolmogorov (K) [13], Cramér–Mises–Smirnov (CMS) [14], Anderson–Darling (AD) [15, 16], Kuiper (Ku) [17], Watson (W) [18, 19] show similar behavior dependent on the degree of roundoff; the same holds for the distributions of the Zhang [20] test statistics which are dependent on the sample size. Let us show it exemplified by the Cramér–Mises–Smirnov criterion with statistic of the form

$$S_\omega = \frac{1}{12n} + \sum_{i=1}^{n} \left\{ F(x_i,\theta) - \frac{2i-1}{2n} \right\}^2 . \tag{2}$$

When the simple hypothesis is valid in the situation where the roundoff errors may be ignored, the distribution of statistic (2) rapidly converges to the limit distribution $a1(s)$ [14] (we may ignore the deviation from the limit distribution already for $n > 25$ [7]).

With the roundoff, as n increases, the distribution of the stastistic begins to deviate from $a1(s)$ [6]. The deviation of the distribution of statistic (2) from $a1(s)$ is presented in [12] at $n = 1000$ in dependence

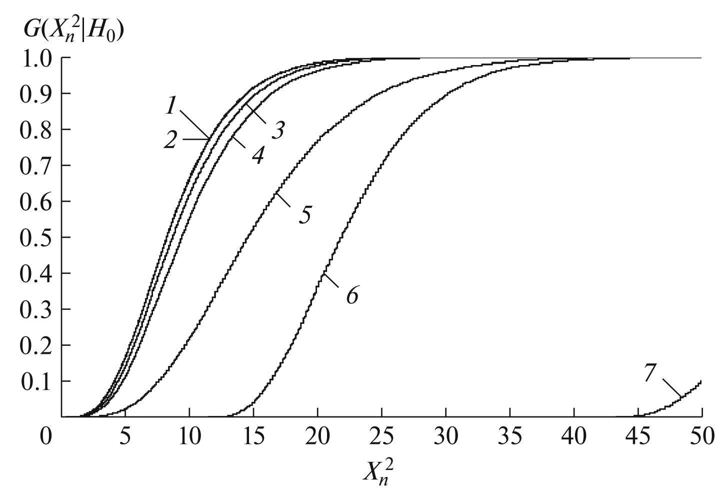

Fig. 1. Dependence of distributions of statistic (1) on Δ at $n = 100$: (*1*) without roundoff, coincides with χ^2_9-distribution; (*2*) at $\Delta = 0.01\sigma$; (*3*) at $\Delta = 0.05\sigma$; (*4*) at $\Delta = 0.1\sigma$; (*5*) at $\Delta = 0.2\sigma$; (*6*) at $\Delta = 0.3\sigma$; (*7*) at $\Delta = 0.5\sigma$.

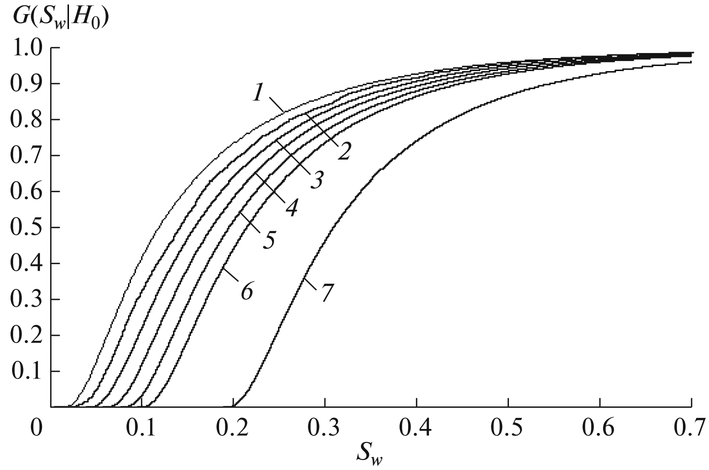

Fig. 2. Dependence of distributions of statistic (2) on n at $\Delta = 0.5\sigma$: (*1*) distribution $a1(S)$; (*2*) for $n = 10$; (*3*) for $n = 20$; (*4*) for $n = 30$; (*5*) for $n = 40$; (*6*) for $n = 50$; and (*7*) for $n = 100$.

on the roundoff error value. When the degree of roundoff Δ is significant, the distribution of statistic (2) may considerably deviate from $a1(s)$ also for rather small n. As a confirmation, in Fig. 2 we show the limit distribution $a1(s)$ of statistic (2) and the empirical distributions $G(S_\omega|H_0)$ of this statistic for different sample sizes n at $\Delta = 0.5\sigma$.

3. EXAMPLE

Let us show how the results of testing a simple hypothesis that the sample 1.05; 1.10; 0.95; 0.90; 0.95; 1.05; 0.95; 0.95; 1.00; 1.05; 1.05; 0.90; 1.00; 1.10; 0.85; 1.10; 1.00; 1.00; 0.95; 1.00; 0.85; 0.95; 0.95; 1.10; 1.10; 1.05; 1.15; 1.10; 0.80; 0.85; 0.95; 1.00; 1.05; 1.00; 1.05; 1.05; 0.95; 1.15; 1.00; 1.15; 0.95; 0.90; 0.95; 0.90; 1.00; 1.20; 1.10; 1.05; 1.00; 1.05 belongs to the normal law with the parameters $\mu = 1$ and $\sigma = 0.1$ vary when the roundoff error is taken into account. The sample is the result of simulating this law with the roundoff error $\Delta = 0.5\sigma$.

We present the results of testing the hypothesis against a set of non-parametric goodness-of-fit tests and Pearson's χ^2 test in Table 1, where we show the values of statistics and the estimates of achieved significance levels p_{value} computed by asymptotic and real distributions of statistics occurring at the roundoff error $\Delta = 0.5\sigma$. In the case of Pearson's χ^2 test we used 5 equally probable intervals. We can see that the estimates of p_{value} over the real distributions of statistics significantly differ from the values obtained by asymptotic distributions.

If the alternative hypothesis H_1 that the sample belongs to some another law is indeed true, then the conditional distributions of statistic $G(S_n|H_1)$ occurring in testing the hypothesis H_0 also vary dependent on Δ. As Δ grows, the test power may either decrease or increase.

As an example, in Table 2 we provide the estimates of powers of goodness-of-fit tests in testing simple and composite (with the estimation of two parameters of the law by the maximum likelihood method) hypotheses that the samples belong to the normal law. As an alternative hypothesis H_1, with respect to which we estimated the power, we consider the logistic law with the density

$$f(x) = \frac{\pi}{\sqrt{3}\theta_1} \frac{\mathrm{e}^{-\pi(x-\theta_0)/(\sqrt{3}\theta_1)}}{[1 + \mathrm{e}^{-\pi(x-\theta_0)/(\sqrt{3}\theta_1)}]^2},$$

which is very close to the normal one.

The estimates of power $1 - \beta$, where β is the probability of type II error (non-rejection of the hypothesis H_0 when the alternative hypothesis H_1 is true), are given at $n = 100$ for the probability of type I error $\alpha = 0.1$. They were obtained with the number of simulation experiments $N = 10^6$, which provides the estimates of power with an accuracy of approximately $\pm 10^{-3}$.

Table 1. Results of hypothesis testing

No.	Test	Statistic	Estimates of p_{value}	
			from asymptotic distribution of statistic	from real distribution of statistic
1	K	1.073889	0.19903	0.721
2	CMS	0.156638	0.36997	0.776
3	AD	0.876389	0.4291	0.847
4	Ku	1.7235144	0.05723	0.791
5	W	0.1430680	0.1187	0.672
6	χ^2	5.8310722	0.21212	0.497

Table 2. Estimates of power of goodness-of-fit tests dependent on Δ

No.	Test	Estimates of power					
		$\Delta = 0$	$\Delta = 0.05\sigma$	$\Delta = 0.1\sigma$	$\Delta = 0.2\sigma$	$\Delta = 0.5\sigma$	$\Delta = \sigma$
Simple hypothesis is tested							
1	K	0.127	0.126	0.133	0.131	0.150	0.240
2	Ku	0.199	0.202	0.209	0.224	0.265	0.340
3	CMS	0.113	0.113	0.114	0.116	0.134	0.211
4	W	0.208	0.209	0.211	0.218	0.254	0.318
5	AD	0.124	0.124	0.124	0.125	0.129	0.148
6	χ^2	0.150	0.149	0.146	0.128	0.078	0.068
Composite hypothesis is tested							
1	K	0.240	0.238	0.233	0.228	0.230	0.333
2	Ku	0.273	0.274	0.274	0.280	0.303	0.340
3	CMS	0.294	0.295	0.297	0.307	0.339	0.318
4	W	0.294	0.295	0.298	0.308	0.340	0.317
5	AD	0.327	0.327	0.328	0.328	0.320	0.256
6	χ^2	0.117	0.118	0.117	0.112	0.091	0.136

We may note that, as the roundoff error Δ increases, the power of χ^2 test decreases, but the power of non-parametric goodness-of-fit tests may increase.

We point out that the roundoff error may similarly affect the distributions of statistics of multiple special criteria oriented on testing the belonging of random values to a specific distribution law (normal [8], uniform [9], exponential, etc.). Works [21, 22] are focused on the mistakes and wrongdoings that may lead to incorrect conclusions in using the goodness-of-fit tests. In view of the above said, it is clear that we should also pay attention to the possible influence of roundoff errors on our conclusions.

4. EFFECT OF THE ROUNDOFF ON DISTRIBUTIONS OF HOMOGENEITY STATISTICS

In k-sample tests two or more samples are simultaneously analyzed. The distributions $G(S|H_0)$ of law homogeneity test statistics, in which the hypothesis of type H_0: $F_1(x) = F_2(x) = \cdots = F_k(x)$ is tested, are influenced by the degrees of roundoff and their difference in the analyzed samples. We consider

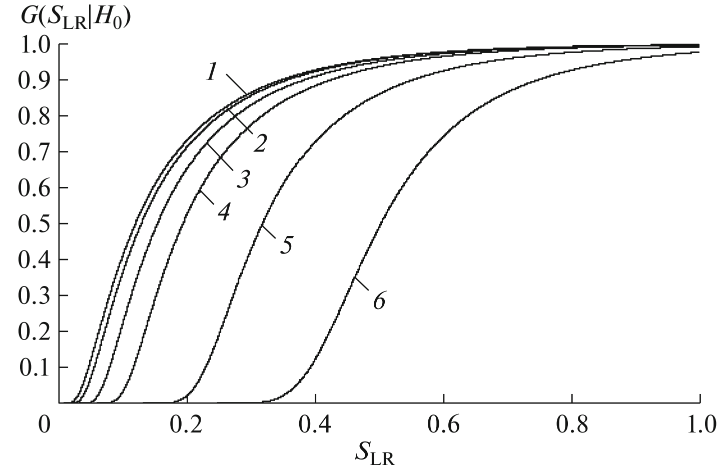

Fig. 3. Dependence of distributions of statistic (3) on Δ_2 at $\Delta_1 = 0.01\sigma$: (*1*) at $\Delta_2 = \Delta_1$ coincides with $a1(s)$; (*2*) at $\Delta_2 = 0.1\sigma$; (*3*) at $\Delta_2 = 0.2\sigma$; (*4*) at $\Delta_2 = 0.3\sigma$; (*5*) at $\Delta_2 = 0.5\sigma$; and (*6*) at $\Delta_2 = 0.7\sigma$.

the effect of the roundoff errors on the distributions of homogeneity test statistics on the example of two-sample Lehmann–Rosenblatt (LR) and Smirnov (Sm) tests.

The statistic of the Lehmann–Rosenblatt test considered in [23, 24] is determiend by

$$S_{\mathrm{LR}} = \frac{1}{n_1 n_2 (n_1 + n_2)} \left[n_1 \sum_{j=1}^{n_1} (s_j - j)^2 + n_2 \sum_{i=1}^{n_2} (r_i - i)^2 \right] - \frac{4 n_1 n_2 - 1}{6(n_1 + n_2)}, \tag{3}$$

where s_j is the index number (rank) of x_{1j}, r_i is the index number (rank) of x_{2i} in the joint variational series of two samples with sizes n_1 and n_2.

When the tested hypothesis H_0: $F_1(x) = F_2(x)$ is true, the limit distribution of statistic (3) is the same distribution $a1(s)$ [24] that becomes limit for the CMS goodness-of-fit test statistic.

We consider the effect of the roundoff error on the distributions of homogeneity statistics when H_0 is true (without loss in generality) in the case when the analyzed samples belong to the standard normal law.

At $\Delta_1 = \Delta_2$, sample sizes $n_i = 100$, and $\Delta_i \leq 0.5\sigma$, the distributions $G(S_{\mathrm{LR}}|H_0)$ do not practically deviate from the distribution $a1(s)$, but they do deviate at unequal Δ_i.

In Fig. 3 the distributions $G(S_{\mathrm{LR}}|H_0)$ of test statistic (3) at $n_i = 100$ are shown in dependence on Δ_2 at $\Delta_1 = 0.01\sigma$. In this case the deviation of the distribution $G(S_{\mathrm{LR}}|H_0)$ from $a1(s)$ at $\Delta_1 = 0.05\sigma$ is yet practically insignificant. At the same Δ_i, as n_i grows, the deviations of $G(S_{\mathrm{LR}}|H_0)$ from $a1(s)$ increase.

The Smirnov homogeneity test was proposed in work [25]. In contrast to the original version [14], we consider the test with the modified statistic of the following form [26]:

$$S_{\mathrm{Sm}} = \sqrt{\frac{n_1 n_2}{n_1 + n_2}} \left(D_{n_1,n_2} + \frac{n_1 + n_2}{4.6 n_1 n_2} \right), \tag{4}$$

where $D_{n_1,n_2} = \max(D^+_{n_1,n_2}, D^-_{n_1,n_2})$,

$$D^+_{n_1,n_2} = \max_{1 \leq r \leq n_1} \left[\frac{r}{n_1} - F_{n_2}(x_r) \right] = \max_{1 \leq s \leq n_2} \left[G_{n_1}(y_s) - \frac{s-1}{n_2} \right],$$

$$D^-_{n_1,n_2} = \max_{1 \leq r \leq n_1} \left[F_{n_2}(x_r) - \frac{r-1}{n_1} \right] = \max_{1 \leq s \leq n_2} \left[\frac{s}{n_2} - G_{n_1}(y_s) \right].$$

The discrete distribution of statistic (4), different from the original one [14] by the presence of second term in parentheses, converges faster to the asymptotic Kolmogorov distribution $\mathrm{K}(S)$ [26]. The set of possible values of statistic is a grid with the step $1/k$, where k is the least common multiplier of n_1

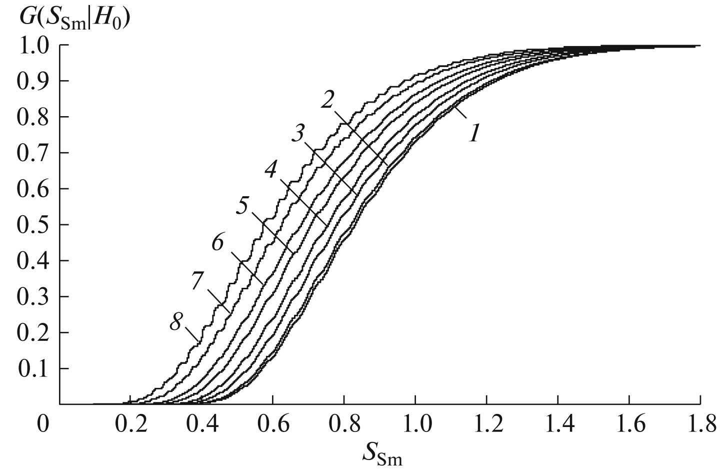

Fig. 4. Dependence of distributions of statistic (4) on Δ_i: (*1*) at $\Delta_i = 0$; (*2*) at $\Delta_i = 0.01\sigma$; (*3*) at $\Delta_i = 0.05\sigma$; (*4*) at $\Delta_i = 0.1\sigma$; (*5*) at $\Delta_i = 0.2\sigma$; (*6*) at $\Delta_i = 0.3\sigma$; (*7*) at $\Delta_i = 0.5\sigma$; and (*8*) at $\Delta_i = 0.7\sigma$.

and n_2 [14]. Therefore, it is more advantageous to apply the test when the sample sizes n_1 and n_2 are not equal and are coprimes.

We can see in Fig. 4 that the distributions $G(S_{\rm Sm}|H_0)$ of statistic (4) vary in a completely different manner in dependence on the degree of roundoff. In the figure we show the distributions $G(S_{\rm Sm}|H_0)$ at $\Delta_1 = \Delta_2$ and sample sizes $n_1 = 101$ and $n_2 = 103$. As the roundoff error grows, the distributions $G(S_{\rm Sm}|H_0)$ shift towards the region of lower values.

In this case the distribution of statistic without roundoff errors of results practically coincides with the Kolmogorov distribution. At the roundoff error $\Delta_i = 0,01\sigma$ the deviation from $\mathrm{K}(S)$ is already visible, and at $\Delta_i = 0.05\sigma$ it cannot be ignored anymore.

The study of the dependence of test powers with statistics (3) and (4) on the degree of roundoff Δ_i (for $0 \leq \Delta_i \leq 0.7\sigma$ and $\Delta_1 = \Delta_2$) showed that the growth in roundoff errors Δ_i does not significantly influence the estimates of power.

Needless to say, the roundoff errors affect the distributions of statistic of the two-sample homogeneity test of Anderson–Darling–Petit [27] as well as the distribution of statistics of the k-sample homogeneity tests of Anderson–Darling [28] and Zhang [29] and of the k-sample tests based on the use of two-sample ones [30].

The study of the distributions of statistics of two- and k-sample parametric homogeneity tests of average, used for checking the hypothesis H_0: $\mu_1 = \mu_2 = \cdots = \mu_k$ or for checking the hypothesis H_0: $\mu = \mu_0$ about the equality of the mathematical expectation to its nominal value, showed that the roundoff errors of measurement results have no significant effect on them.

At the same time, the distributions of statistics of parametric criteria, applied for testing similar hypotheses with respect to variances H_0: $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_k^2$ and H_0: $\sigma^2 = \sigma_0^2$, may be significantly influenced by the roundoff errors. In particular, the distributions $G(S|H_0)$ of statistics of the parametric Bartlett test [31] and Cochran C-test [32] vary if the roundoff errors Δ_i of the compared samples are different. And the powers of these tests, as well as the power of the non-parametric Klotz test [33], decrease with increasing Δ_i even at equal Δ_i [12].

5. CONCLUSIONS

In the situations when the roundoff error Δ_i appears to be comparable with the standard deviation σ of the distribution law of measurement error, the real distributions $G(S_n||H_0)$ of test statistics for checking statistical hypotheses at restricted sample sizes may significantly deviate from the limit distributions $G(S|H_0)$ of these statistics or from $G(S_n|H_0)$ occurring in the classical case (where the roundoff errors can be ignored). In such situations the classical results (the asymptotic distributions of statistics and the tables of critical values) related with the test properties cannot be applied. Neglect of this fact will lead (more often) to an increase in type I errors (rejection of true hypothesis H_0) or (less often, see the Smirnov homogeneity test) to an increase in type II errors (non-rejection of H_0 when some alternative hypothesis is true).

The possible effect of the roundoff errors on the distributions of test statistics must be taken into account in using specific tests in applications and in the automated systems of data processing, where the statistical methods may be used to track persistence of laws (or detect their change).

Variation in test properties under the influence of roundoff errors does not exclude its possible correct application. We just need to know the distribution $G(S_n|H_0)$ of test statistic at the same roundoff errors Δ_i and same sample sizes n_i which correspond to the analyzed samples. For this purpose the best thing is to use statistical modeling methods. To simulate $G_N(S_n|H_0)$ of multiple tests considered in [7–10], we may use the program system ISW [11], which was applied to carry out the current investigations and which includes the corresponding means of interactive modeling. The similar capabilities may be embedded in any other system of data processing.

Finally, we underline that the signal for cautiousness in applying the classical results with respect to the used tests is the presence of too many repeated values. If this is not the case, then we may rely on the classical results. Otherwise, we should follow the proposed recommendations or decline to use the corresponding test.

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