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APPLICATION AND POWER OF PARAMETRIC CRITERIA FOR TESTING THE HOMOGENEITY OF VARIANCES. PART IV

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The application of parametric criteria for testing the homogeneity of variances (Bartlett, Cochran, Fisher, Hartley, Levene, Neyman–Pearson, O'Brien, Link, Newman, Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, the Overall–Woodward Z-test and modified Z-test) is examined, including under conditions such that the standard assumption of normality is violated. A comparative analysis is made of the power of the criteria. The permissible level of significance is estimated for the case in which the distributions of the statistics for the applied tests are unknown.

Keywords: *test, statistic, homogeneity of variances, distribution of a statistic, power of tests.*

The distributions of the statistics of parametric criteria for the homogeneity of variances are studied in [1], which is an extension of [2, 3]: Neyman–Pearson [4], O'Brien [5], Link (ratio of ranges) [6], Newman (studentized range) [7], Bliss–Cochran–Tukey [8], Cadwell–Leslie–Brown [9], Overall–Woodward *Z*-test [10], and the modified Overall–Woodward *Z*-test [11]. A comparative analysis of the power of tests is given [1], including estimates of the power for parametric (Bartlett [12], Cochran [13], Fisher [14], Hartley [15], Levene [16]) and nonparametric (Ansari–Bradley [17], Mood [18], Siegel–Tukey [19]) tests. Descriptions are given for the statistics of all these tests that are intended for testing hypotheses of the homogeneity of the variances of m samples H_0: $\sigma_1^2 = \sigma_2^2 = ... = \sigma_m^2$. The competing hypothesis is usually taken to be H_1: $\sigma_{i_1}^2 \neq \sigma_{i_2}^2$, where the inequality is satisfied for at least one pair of indices i_1, i_2. These studies showed that:

1. Parametric tests (in very case, the best representatives of these) have a clear advantage in power compared to the nonparametric tests.

2. The standard assumption validating the use of parametric tests for the homogeneity of variances is that the samples to be analyzed adhere to a normal distribution law. When this assumption is violated, there are significant changes in the distributions of the test statistics corresponding to validity of the hypothesis H_0. This excludes the possibility of using the classical results obtained under the assumption of normality. The exception is the group of stable criteria (O'Brien, Levene, and the modified Overall–Woodward *Z*-test). But in these cases, as well, the dependence on the form of the distribution of the sample being analyzed was also traced.

3. Even if the standard assumption is satisfied, the possibility of correct application of a number of parametric tests is limited, since the distributions of the statistics are unknown and during hypothesis testing it is necessary to rely of tables of critical values for some series of sample volumes, so it is impossible to estimate the attained significance level p.

4. For limited sample volumes, the distributions of the statistics of parametric tests often differ substantially from the known asymptotic distributions of these statistics which occur when the standard assumption is satisfied.

5. The assumption of normality is not imposed on nonparametric criteria in which a hypothesis of equality of scale parameters is being tested. However, an equally strong assumption about the homogeneity of the distributions of the samples being analyzed must be satisfied [3].

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6. The distributions of the normalized statistics of the nonparametric tests (Ansari–Bradley, Mood, Siegel–Tukey) are discrete. For small sample volumes, they differ substantially from the standard normal distribution used to describe them.

7. The parametric tests have an obvious advantage in power over the nonparametric distributions, even when the samples have non-normal distributions [3].

Dependence of the power of criteria on the form of the distribution. When the standard assumption of normality of distributions is violated, the power of the criteria was studied for situations in which the samples adhere to a generalized normal distribution with the density

$$f(x) = \frac{\theta_0}{2\theta_1\Gamma(1/\theta_0)}\exp\left[-\left(|x-\theta_2|/\theta_1\right)^{\theta_0}\right], \tag{1}$$

where the shape parameter θ_0 takes different values.

Special cases of this family of distributions include the normal distribution with $\theta_0 = 2$ and the Laplace distribution with $\theta_0 = 1$. Estimates of the power of criteria when the samples have generalized normal distributions (1) with different values of θ_0 for sample sizes $n_i = 100, i = 1, ..., m$, are listed in Tables 1 and 2. The notation $De(\theta_0)$ corresponds to a distribution of form (1) with different values of the shape parameter θ_0. When the shape parameter θ_0 is smaller, the tails of the distribution $De(\theta_0)$ will be heavier, and *vice-versa*, when that parameter is larger, the tails will be lighter. The competing hypothesis H_1 was taken to be the situation where $m - 1$ samples adhere to a distribution with $\sigma = \sigma_0$ and sample number m has a distribution with variance $\sigma_m = 1.5\sigma_0$.

The criteria are ranked in order of decreasing power when they are applied to a normal distribution (see Table 1 for $De(2)$). The maximum powers obtained for a given distribution are indicated in bold face. The power estimates show that the order of preference for the criteria changes with the heaviness of the tails. It can be seen that the Bartlett, Cochran, Hartley, Fisher, Neyman–Pearson, and the Overall–Woodward *Z*-tests remain equivalent in terms of power in situations in which the standard assumption of normality fails and the two samples being analyzed have symmetric distributions. Similarly, the Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, and Link group of criteria remain equivalent in terms of power.

If the samples have distributions with lighter (compared to a normal distribution) tails, then ordering of the criteria in terms of power is the same as for a normal distribution.

For (symmetric) distributions with heavier tails than normal, the order of preference changes. In the case of heavy tails (see Table 1 for $De(0.5)$), the criteria are ranked as follows:

Mood ≻ Levene ≻ Siegel–Tukey ~ Ansari–Bradley ≻ O'Brien ≻ Modified Z-test ≻ group of criteria (Bartlett, Cochran, Hartley, Fisher, Neyman–Pearson, Overall–Woodward Z-test) ≻ group of criteria (Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, Link).

It should be noted that as the heaviness of the tails increases, the power of all these parametric criteria decreases. The Newman, Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, and Link criteria have the advantage over the nonparametric criteria for arbitrary observed distributions (including for $n_i = 100$), but only for very light tails of the observed distributions. When the number of compared samples increases, the situation changes (see Table 2). The groups of equivalent criteria essentially vanish. The sole exceptions are the Bartlett and Neyman–Pearson criteria, which remain equivalent in power in any situation.

In the case of heavy tails, in terms of decreasing power the criteria are ordered as follows:

Levene ≻ O'Brien ≻ Modified Z-test ≻ Bartlett ~ Neyman–Pearson ≻ Overall–Woodward Z-test ≻ Hartley ≻ Cochran ≻ Cadwell–Leslie–Brown ≻ Bliss–Cochran–Tukey.

The results of this comparison can be supplemented by estimates of the power of the power of the Klotz [20] and Fligner–Killeen [21] criteria, which are discussed in [22].

Calculating the attained level of significance. Hypothesis H_0 testing based on the attained level of significance p is always better justified and more informative than a comparison of the calculated value of a statistic S^* with an indication of

TABLE 1. Power of the Criteria with Respect to H_1 for $m = 2$

Criterion	*De*(0.5)			*De*(2)			*De*(5)		
	Significance level α								
	0.100	0.050	0.010	0.100	0.050	0.010	0.1	0.05	0.01
Bartlett, Cochran, Hartley, Fisher, Neyman–Pearson, *Z*-tests	0.162	0.091	0.022	**0.564**	**0.438**	**0.218**	**0.791**	**0.689**	**0.446**
Modified *Z*-test	0.167	0.096	0.024	0.555	0.427	0.205	0.790	0.688	0.446
O'Brien	0.176	0.103	0.029	0.555	0.427	0.205	0.782	0.674	0.420
Link	0.215	0.132	0.040	0.515	0.388	0.180	0.649	0.524	0.283
Mood	**0.222**	**0.138**	**0.043**	0.468	0.344	0.152	0.659	0536	0.298
Siegel–Tukey, Ansari–Bradley	0.213	0.131	0.041	0.405	0.287	0.119	0.542	0.416	0.204
Newman	0.144	0.080	0.020	0.386	0.276	0.116	0.720	0.608	0.370
Bliss–Cochran–Tukey, Cadwell–Leslie–Brown, Link	0.128	0.069	0.016	0.289	0.190	0.068	0.650	0.527	0.292

TABLE 2. Power of the Criteria with Respect to H_1 for $m = 5$

Criterion	*De*(0.5)			*De*(2)			*De*(5)		
	Significance level α								
	0.100	0.050	0.010	0.100	0.050	0.010	0.100	0.050	0.010
Cochran	0.134	0.070	0.015	**0.624**	**0.515**	**0.316**	**0.869**	**0.807**	**0.643**
O'Brien	0.160	0.092	0.026	0.575	0.460	0.258	0.815	0.731	0.533
Z-test	0.141	0.074	0.016	0.565	0.445	0.241	0.811	0.722	0.512
Modified *Z*-test	0.148	0.081	0.019	0.554	0.433	0.228	0.810	0.721	0.514
Bartlett, Neyman–Pearson	0.142	0.075	0.016	0.557	0.434	0.227	0.806	0.713	0.495
Hartley	0.140	0.074	0.016	0.545	0.418	0.204	0.799	0.699	0.459
Levene	**0.197**	**0.119**	**0.036**	0.513	0.390	0.197	0.657	0.542	0.323
Bliss–Cochran–Tukey	0.114	0.059	0.013	0.262	0.170	0.061	0.704	0.601	0.384
Cadwell–Leslie–Brown	0.119	0.062	0.013	0.253	0.158	0.052	0.638	0.513	0.280

the critical values taken from the corresponding table of percentage points. In the latter case, it is not clear how well justified this decision may be. In the case of a right-sided criterion, p is defined by

$$p = P\{S > S^* | H_0\} = 1 - G(S^* | H_0), \tag{2}$$

where $G(S|H_0)$ is the probability distribution function of the statistic for the applied test when H_0 is true.

In the case of a two-sided criterion, the critical region consists of two parts and p is given by

$$p = 2\min\left\{G(S^* | H_0),\ 1 - G(S^* | H_0)\right\}. \tag{3}$$

It is not difficult to calculate p using Eqs. (2) or (3) when the distribution of the test statistic is known. If there is no information about the distribution of the test statistic and only tables of percentage points are available or the sample volumes are relatively small and such that the distribution of the statistic differs substantially from the (asymptotic) limit, then a correct calculation of p presents some problems.

The distributions of most parametric tests for the homogeneity of variances (even in the classical situation) depend substantially on the sample volumes. Thus, for testing a hypothesis H_0 it is necessary to rely on tables of percentage points formulated for bounded sets n_i, which often assume equality of the volumes of the samples being compared. However, even in these situations the quality of the statistical conclusions can be improved by finding an estimate for p.

At present, because of the improved computational techniques in statistical analysis programs, the role of computer methods in studying statistical behavior has expanded. If the distribution of a test statistic used for testing some hypothesis is unknown (for given volumes n_i) at the start of the test (for various reasons), then it is possible to study the distribution of the statistic in real time, i.e., in an interactive mode [23–25]. Then, for example, it is possible to study the unknown distribution of the statistic of any test for the homogeneity of variances that depends on the sample volume for those values of n_i which correspond to the analyzed samples, and to use the empirical distribution of the statistic found by modeling to estimate the attained level of significance. With this approach, the empirical distribution $G_N(S_n|H_0)$ of the statistic for the corresponding test needed for testing the hypothesis is constructed by statistical modeling to an accuracy that depends on the number of runs N in a Monte-Carlo method [26]. Then, the empirical distribution $G_N(S_n|H_0)$ and the value S^* of the test statistic given by Eq. (2) or (3) are used to estimate p. Thus, the results of the statistical simulations in the course of the analysis are used to evaluate progress in the testing of the hypothesis. An interactive mode requires the development of a program with parallel processing for speed in the modeling and drawing on accessible computing resources [27]. With parallel processing, the time to construct the distribution $G_N(S_n|H_0)$ of the test statistic is insignificant against the background of a complete solution of the analysis problem.

Tests under nonstandard conditions. An interactive mode for studying the distributions of statistics makes it possible to use tests under conditions such that the standard assumption that the measurement data follow a normal distribution is violated. A deviation from normality leads to significant changes in the distribution $G(S|H_0)$ of the statistics for testing the homogeneity of variances. This applies to a lesser extent to the O'Brien test, the modified Overall–Woodward Z-test, and the modified Levene test. This stability of the criteria is "paid for" by a slight reduction in power. Despite their stability, the distributions $G(S|H_0)$ of the statistics for these tests deviate so far from those under the standard assumptions that this cannot be ignored when the samples have distributions with heavy tails. Thus, the correctness of the conclusions depends on the accuracy with which the distribution $G(S|H_0)$ corresponding to the actual measurement conditions is known.

We now examine the use of an interactive mode for studying $G(S|H_0)$ and the accuracy with which the level of significance p for different criteria for testing the homogeneity of variances can be estimated depending on the number N of runs in a simulation of empirical distributions of the statistics, including when the standard assumption of normality is violated. In order for the error in estimating p not to exceed 0.01 with confidence coefficient of 0.99, the number N of runs must be on the order of 16600, and for the error to be no more than 0.001, N must be on the order of 1,660,000 [26].

Example 1. Let the test hypothesis be equality of the variances of two successive samples of volume $n_i = 40$, $i = 1, 2$, assuming that they have normal distributions:

0.205	0.232	–0.219	0.829	0.127	0.939	0.995	0.706	–0.450	–0.361
–0.364	–0.107	1.054	–0.095	–2.188	0.453	–1.052	0.640	–0.417	–2.144
–3.473	–0.857	–0.678	0.070	–1.139	0.574	0.409	0.206	0.184	1.273
–0.326	–1.245	0.227	0.185	0.383	0.126	0.255	1.110	–0.310	–0.178

0.269	–0.187	–0.013	–1.248	–0.247	–0.541	1.209	–2.814	0.575	–0.452
–0.427	0.337	1.138	–1.090	–0.858	–0.006	–1.212	–0.180	1.751	–0.485
–0.779	–0.752	0.342	–0.175	0.509	0.209	0.596	1.869	1.764	1.084
0.995	0.633	0.003	–0.642	–1.225	–0.115	–1.543	0.137	–1.290	2.189

TABLE 3. Estimates of the Level of Significance p for Analysis of Two Samples with Valid Hypothesis H_0

Criterion	Value of statistic	p			
		Normal distribution			Laplace distribution
		theoretical estimate	$N = 10^4$	$N = 10^6$	$N = 10^6$
Bartlett	0.268028	0.604658	0.605	0.6045	0.734
Cochran	0.541643	–	0.605	0.6045	0.734
Fisher	0.846236	0.604671	0.596	0.6045	0.734
Hartley	1.181700	–	0.605	0.6045	0.734
Neyman–Pearson	1.003490	0.607000	0.605	0.6045	0.734
Z-test	0.279266	0.597183	0.605	0.6045	0.734
Modified *Z*-test	0.115111	0.734398	0.741	0.7348	0.732
O'Brien	0.162623	0.687856	0.702	0.6971	0.722
Levene	0.604953	–	0.454	0.4451	0.459
Newman	4.564110	–	0.780	0.7777	0.730
Link	0.948631	–	0.806	0.8079	0.877
Bliss–Cochran–Tukey	0.513181	–	0.814	0.8084	0.878
Cadwell–Leslie–Brown	1.054150	–	0.814	0.8084	0.878

Table 3 lists the values of the statistics calculated during testing of the homogeneity of the variances corresponding to these two samples and gives the estimates of p for the simulated distributions of the statistics of the test for $N = 10^4$ and $N = 10^6$ assuming a normal distribution for the random quantities. For the tests with known asymptotic distributions of the statistics, the table also lists theoretical estimates of p calculated according to these distributions.

In this case, the test hypothesis H_0 is true, but both samples were simulated using a Laplace distribution with $\sigma = 1$. The last column (Table 3) shows the estimates of p calculated using the distributions of the test statistics for $N = 10^6$ under the assumption that the random quantities have a Laplace distribution. There is a significant difference in the estimates of p for the Laplace and normal distributions, but for the stable Levene and O'Brien tests, and the modified *Z*-test, this difference is minimal.

If the actual distribution has heavier tails than the normal distribution and when the classical results associated with normality can be taken into account when using a parametric criterion for the homogeneity of variances, then this leads to an increased (compared to the given amount) probability of a type I error and a reduction in the probability of a type II error. If the actual distribution has lighter tails, then in the analogous situation this leads to a reduction in the probability of a type I error and an increase in the probability of a type II error.

Example 2. We now test the hypothesis of equal dispersions for three samples, two of which are taken from the previous example, while the second is listed below:

0.254	–0.254	–0.017	0.002	1.937	–2.476	–0.092	–0.543	2.588	1.970
1.869	0.453	–0.616	–2.806	2.382	0.476	0.641	–2.581	–0.659	–0.027
1.775	2.154	–1.801	–0.774	–0.522	1.413	–0.042	–0.175	–0.929	0.664
–0.298	0.409	0.040	0.418	0.478	–0.052	–4.354	1.521	–2.126	1.177

This sample was also modelled with a Laplace distribution but for $\sigma = 1.5$.

Table 4 lists the values of the statistics calculated during testing of the hypothesis of homogeneity of the variances corresponding to these three samples. When it is assumed that the samples have normal distributions, theoretical values of p

TABLE 4. Estimates of the Level of Significance p for an Analysis of Three Samples ($\sigma_1:\sigma_2:\sigma_3$ = 1:1:1.5)

Criterion	Value of statistic	p		
		Normal distribution		Laplace distribution
		Theoretical estimate	$N = 10^6$	$N = 10^6$
Bartlett	9.729430	0.0077	0.0079	0.1198
Cochran	0.534165	–	0.0032	0.0667
Hartley	2.501720	–	0.0140	0.1508
Neyman–Pearson	1.087740	–	0.0079	0.1198
Z-test	5.005400	0.0067	0.0065	0.1098
Modified Z-test	2.315710	0.0987	0.0953	0.0897
O'Brien	3.224100	0.0434	0.0396	0.0336
Levene	2.824730	–	0.0661	0.0713
Bliss–Cochran–Tukey	0.415913	–	0.0861	0.3301
Cadwell–Leslie–Brown	1.462710	–	0.1899	0.5106

are given, as well as estimates calculated from the statistical simulation of the distributions of the statistics for $N = 10^6$. Only the estimates of p for $N = 10^6$ are listed when a Laplace distribution is assumed. These results show that if we ignore the fact that the standard assumption of normality is violated, then for all the tests (except the modified Z-test) we obtain values of p that are less than the true values for a Laplace distribution. If the actual distribution had lighter tails than the normal distribution, then the values of p found under the assumption of normality would exceed the true values.

The result for the modified Overall–Woodward Z-test does not fit into the above remarks (Table 4). This is because of the approximate character of the structure of the modification [1]. These examples show that the parametric tests for the homogeneity of variances with the highest powers can be used correctly, both when the standard assumption of normality is satisfied and when it is violated. In both cases, the possibility of estimating the attained levels of significance enhances the information content of the statistical conclusions.

This sort of procedure for using the tests is possible with reliance on programs similar to that in Ref. 24. One of the preliminary conditions for switching to this procedure is identification of the form of the distribution which provides the best description of the sample being analyzed [28]. A decision on the most preferable model for the distribution may lie beyond the scope of the problem of testing for homogeneity. Otherwise, a choice of the model for the distribution may be made in the course of an analysis of a set of test samples (or a unified sample). If the best model is then a family of distributions (e.g., a generalized normal distribution, the family of gamma- and beta-distributions, etc.) for which the specific form of the distribution is determined by a shape parameter (or parameters), then the distribution model that is used must be identified with an accuracy in this parameter (an estimate must be found for it and set).

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REFERENCES

1. B. Yu. Lemeshko and T. S. Sataeva, "Application and power of parametric criteria for testing the homogeneity of variances, Part 3," *Izmer. Tekhn.*, No. 1, 8–13 (2017).
2. B. Yu. Lemeshko, S. B. Lemeshko, and A. A. Gorbunova, "Application and power of criteria for testing the homogeneity of variances. Part I. Parametric criteria," *Izmer. Tekhn.*, No. 3, 10–16 (2010).

3. B. Yu. Lemeshko, S. B. Lemeshko, and A. A. Gorbunova, "Application and power of criteria for testing the homogeneity of variances. Part 2. Nonparametric criteria," *Izmer. Tekhn.,* No. 5, 11–18 (2010).
4. A. I. Kobzar', *Applied Mathematical Statistics for Engineers and Scientific Workers*, FIZMATLIT, Moscow (2006).
5. R. G. O'Brien, "Robust techniques for testing heterogeneity of variance effects in factorial designs," *Psychometrika*, **43**, No. 3, 327–342 (1978).
6. R. F. Link, "The sampling distribution of the ratio of two ranges from independent samples," *Ann. Math. Stat.*, **21**, No. 1, 112–116 (1950).
7. D. Newman, "The distribution of range in samples from a normal population, expressed in terms of an independent estimate of standard deviation," *Biometrika*, **31**, 20–30 (1939).
8. C. I. Bliss, W. G. Cochran, and J. W. Tukey, "A rejection criterion based upon the range," *Biometrika*, **43**, 418–422 (1956).
9. R. T. Leslie and B. M. Brown, "Use of range in testing heterogeneity of variance," *Biometrika*, **53**, 221–227 (1966).
10. J. E. Overall and J. A. Woodward, "A simple test for heterogeneity of variance in complex factorial design," *Psychometrica*, **39**, 311–318 (1974).
11. J. E. Overall and J. A. Woodward, *A Robust and Powerful Test for Heterogeneity of Variance*, Univ. of Texas Med. Branch Psychometric Lab. (1976).
12. M. S. Bartlett, "Properties of sufficiency of statistical tests," *Proc. Roy. Soc. A.*, **160**, 268–287 (1937).
13. W. G. Cochran, "The distribution of the largest of a set of estimated variances as a fraction of their total," *Ann. Eugenics*, **11**, 47–52 (1941).
14. L. N. Bol'shev and N. V. Smirnov, *Tables of Mathematical Statistics*, Nauka, Moscow (1983).
15. H. O. Hartley, "The maximum F-ratio as a short-cut test of heterogeneity of variance," *Biometrika*, **37**, 308–312 (1950).
16. H. Levene, "Robust tests for equality of variances," *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling* (1960), pp. 278–292.
17. A. R. Ansari and R. A. Bradley, "Rank-tests for dispersions," *Ann. Math. Stat.*, **31**, No. 4, 1174–1189 (1960).
18. A. Mood, "On the asymptotic efficiency of certain nonparametric tests," *Ann. Math. Stat.*, **25**, 514–522 (1954).
19. S. Siegel and J. W. Tukey, "A nonparametric sum of rank procedure for relative spread in unpaired samples," *J. Amer. Stat. Ass.*, **55**, No. 291, 429–445 (1960).
20. J. Klotz, "Nonparametric tests for scale," *Ann. Math. Stat.*, **33**, 498–512 (1962).
21. M. A. Fligner and T. J. Killeen, "Distribution-free two-sample tests for scale," *J. Amer. Stat. Ass.*, **71**, No. 353, 210–213 (1976).
22. B. Yu. Lemeshko, *Criteria for Testing the Deviation of a Distribution from Normal. Guidance for Applications*, INFRA-M, Moscow (2015), DOI 10.1273/6086.
23. A. A. Gorbunova and B. Yu. Lemeshko, "Application of variance homogeneity tests under violation of normality assumption," in: *Proc. lnt. Workshop Applied Methods of Statistical Analysis. Simulations and Statistical Inference, AMSA'2011*, Novosibirsk, Russia, Sept. 20–22, 2011, pp. 28–36.
24. B. Yu. Lemeshko, S. B. Lemeshko, and A. P. Rogozhnikov, "Real-time studying of statistic distributions of non-parametric goodness-of-fit tests when testing complex hypotheses," *ibid.*, pp. 19–27.
25. B. Yu. Lemeshko, A. A. Gorbunova, S. B. Lemeshko, and A. P. Rogozhnikov, "Application of nonparametric goodness-of-fit tests for composite hypotheses in case of unknown distributions of statistics," in: *Proc. Int. Workshop Applied Methods of Statistical Analysis. Applications in Survival Analysis, Reliability and Quality Control, AMSA'2013*, Novosibirsk, Russia, Sept. 25–27, 2013, pp. 8–24.
26. B. Yu. Lemeshko and S. N. Postovalov, *Computer Technologies for Data Analysis and Studies of Statistical Behavior: Textbook*, NGTU, Novosibirsk (2004).
27. *ISW – a Program System for Statistical Analysis of One-Dimensional Random Quantities*, www.ami.nstu.ru/~headrd/ISW.htm. acc. May 23, 2016.
28. B. Yu. Lemeshko, "On the identification of a distribution law for the random component of measurement error," *Metrologiya*, No. 7, 8–17 (2004).