Tests for an Absence of Trend[1]

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Abstract

The properties of various parametric and nonparametric tests are studied using methods of statistical simulation. Such tests are designed to test hypotheses for randomness or absence of a trend in dispersion characteristics. Statistics distributions and the test powers are studied with respect to various competitive laws. Advantages and disadvantages of the studied tests are noted.

The procedure of interactive simulation of distributions of the test statistics is proposed and implemented. Such procedure allows making valid conclusions when using the test in the case of violation of standard assumptions.

Keywords: trend, hypothesis of randomness, statistical simulation, test power.

Introduction

A variety of parametric and nonparametric tests has been proposed at different times to test the hypothesis for randomness or absence of a trend in the mathematical expectation and in the dispersion characteristics. However, available sources do not allow us to judge the benefits of a particular test and do not contain any distinct recommendations on the area of application and prerequisites providing correctness of statistical conclusions when using the tests under consideration.

As a rule, assumption of normal distribution law of noise is the main prerequisite for ensuring the correct application of parametric tests, but it is not always realized in practice. The usage of nonparametric tests is based on asymptotic distribution of statistics of such tests. For limited sample sizes, the distributions of statistics of parametric and non-parametric tests may differ significantly from the corresponding limit distributions of statistics used for testing the hypothesis. The common disadvantage of nonparametric tests is an apparent discreteness of the statistics distribution. In such situations, the usage of the limiting (asymptotic) distribution of the statistics instead of the actual distribution of such statistics to test the hypothesis may lead to wrong conclusion.

In this paper, the methods of statistical simulation are used to investigate the statistic distributions and the power of tests for an absence of trend in a mathematical expectation, as well as the dispersion characteristics of the observed random variables.

When testing the absence of a trend in the mathematical expectation, it is assumed that time series of values $x_1, x_2, ...x_n$ of mutually independent random variables with mathematical expectations $m_1, m_2, ...m_n$ and equal (but unknown) variances are

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observed. The hypothesis $H_0 : m_i = m, i = 1, 2, ..., n$ is tested that all sample values belong to the same population with mean m, against a competitive hypothesis about the presence of a trend $H_j : |m_{i+1} - m_i| > 0, i = 1, 2, ..., n - 1$.

When testing the absence of a trend in dispersion characteristics, the hypothesis $H_0 : s_i = s, i = 1, 2, ..., n$ is tested that all sample values belong to the same population with standard deviation s, against a competitive hypothesis for the presence of a trend $H_l : |s_{i+1} - s_i| > 0, i = 1, 2, ..., n - 1$.

When testing the absence of a variance shift (in dispersion characteristics) the hypothesis $H_0 : s_1^2 = ... = s_n^2 = s_0^2$ (s_0^2 being unknown) is tested against a competitive hypothesis

$$H_l : s_1^2 = s_2^2 = ... = s_k^2 = s_0^2; s_{k+1}^2 = ... = s_n^2 = s_0^2 + d; (d > 0),$$

for variance value changes in some unknown point (k unknown $1 \le k \le n - 1$).

1 Tests for an absence of trend in mathematical expectation research results

We have carried out the research of statistics distribution and the powers of parametric tests, which are used for testing the hypotheses of a trend absence in mathematical expectation (Autocorrelation test [1], Autocorrelation test modification [1], Dufor-Roy test [2], Ljung-Box test [3], Moran test [4], Wald-Wolfowitz test [5]), as well as non-parametric tests used for the same purposes (Wald-Wolfowitz rank test [5], Dufor-Roy rank test [2], Bartels test [6], Foster-Stewart test [7], Cox-Stuart test [8], Hollin test [16], Wald-Wolfowitz series test [5], Inversion test [9], Cumulative sum test [10, 11], series Wald-Wolfowitz test [5], series Ramachandran-Ranganathan test [12] and number of sign series of the first-order differences [13]).

The results of such research are briefly summarized in Table 1. The tests studied are arranged in the order of power decreasing. Table 1 shows main advantages and disadvantages of tests, noted during the research.

2 Tests for an absence of trend in dispersion characteristics

Statistical distributions and powers of non-parametric tests (Foster-Stewart test [7], Cox-Stuart test [8], Savage test [14, 12], Klotz test [14, 12])and parametric test (Hsu test [15]) which are used to test an absence of trend in dispersion characteristics, are studied here in more detail.

2.1 Foster-Stuart test

This nonparametric test can be used to test hypotheses of absence of a trend in the mean values or in the variances (dispersion characteristics) depending on the used statistics type. The test for an absence of trend in distribution characteristics is given by [7]:

$$S = \sum_{i=2}^{n} S_i, \tag{1}$$

where $S_i = u_i + l_i$;
$u_i = 1$, if $x_i > x_{i-1}, x_{i-2}, ...x_1$, otherwise $u_i = 0$;
$l_i = 1$, if $x_i < x_{i-1}, x_{i-2}, ...x_1$, otherwise $l_i = 0$.
It is clear that $0 \le S \le n - 1$.
In the absence of a trend the normalized statistics

$$\tilde{t} = \frac{S - \mu}{\hat{\sigma}_S}, \tag{2}$$

where

$$\mu = 2\textstyle\sum_{i=2}^{n} \frac{1}{i}, \hat{\sigma}_S = \sqrt{\mu - 4\textstyle\sum_{i=2}^{n} \frac{1}{i^2}} \approx \sqrt{2\ln n - 3.4253},$$

are approximately described by Student's distribution with $\nu = n$ degrees of freedom. The hypothesis of absence of a trend is rejected at large modulus values of statistics (2).

Actually, the area of discrete values is the range of definition of $\tilde{t}$ statistics. The analysis of statistics distributions shows that even with relatively large sample sizes (around $n = 100, 200$) the discrete distributions of test statistics are significantly different from the Student distribution with n degrees of freedom [17, 18]. It follows that the use of achieved significance level (p-value) for calculations instead of the actual (discrete) distributions of statistics of asymptotic Student t-distributions can lead to serious errors.

2.2 Cox-Stuart test

Cox-Stuart test [8] for the hypothesis of an absence of a trend in variance (in dispersion characteristics) is designed as follows.

Initial sample $x_1, x_2, ...x_n$ is divided into $[n/k]$ subsamples with k number of elements $x_1, ..., x_k; x_{k+1}...x_{2k}; x_{2k+1}...x_{3k}; ...; x_{n-k+1}...x_n$ (if n is not divided by k, then the required number of measurements in the center is dropped out). For every ith subsample the range w_i is found $((1 \le i \le r, r = [n/k]))$. Then, the resulting sequence of ranges is tested against the trend in the mean values using the test with statistics

$$S_1^* = \frac{S_1 - E[S_1]}{\sqrt{D}[S_1]}, \tag{3}$$

where

$$S_1 = \textstyle\sum_{i=1}^{[n/2]} (n - 2i + 1)h_{i,n-i+1}, E[S_1] = \frac{n^2}{8}, D[S_1] = \frac{n(n^2-1)}{24},$$

where $h_{i,j} = 1$, if $x_i > x_j$ and $h_{i,j} = 0$, if $x_i \leq x_j (i < j)$. If the hypothesis for the absence of a trend is true, distribution (3) can be approximately described by the standard normal law.

It is recommended to choose the value of k in [8] according to the following correlations:

$$n \geq 90 \rightarrow k = 5; 64 \leq n < 90 \rightarrow k = 4;$$
$$48 \leq n < 64 \rightarrow k = 3; n < 48 \rightarrow k = 2.$$

The discreteness of the S_1^* statistics distribution upon detection of a trend in the variance is significantly higher than the discreteness of the Cox-Stuart statistics distribution for trend in mean. This is natural because the analyzed range sample contains only $[n/k]$ number of elements. When using the Cox-Stuart test for detection of a trend in the dispersion, the difference of statistics discrete distribution from the standard normal law can almost be neglected only for $n > 170$ [19].

2.3 Hsu test for an absence of variance shift and shift point detecting

Under this test the rejection of the hypothesis of randomness (for absence of a trend) can show the discovery of a variance shift. Hsu test statistics are given by [15]

$$H = \frac{\sum_{i=1}^{n} (i-1)(x_i - m_x)^2}{(n-1)\sum_{i=1}^{n} (x_i - m_x)^2}, 0 \leq H \leq 1, \tag{4}$$

where m_x is median of variation series. Under the assumption that the mathematical expectation of a sequence of random variables has the same value, the hypothesis of a constant variances is tested. As a competitive hypothesis, the change in the dispersion of observed values at some (unknown) time (starting from some element of the sample) can be considered. The test is two-sided: the tested hypothesis of absence of a variance shift is rejected for small and large values of the statistics (4).

Usually the test is used in a normalized form

$$H* = \frac{H - 1/2}{\sqrt{D}[H]}, where D[H] = \frac{n+1}{6(n-1)(n+2)}. \tag{5}$$

Under the validity of the hypothesis of the absence of variance changes, statistic (5) obeys the standard normal law asymptotically.

The simulation results [17] show that for $n > 30$ statistics distribution agrees well with the standard normal law.

Statistics distribution (5) strongly depends on the law of distribution to which random variables belong. The greatest deviation from the standard normal law is observed in the case when random variables belong to the laws with heavy tails. Asymmetry of the law significantly affects the statis-tics distribution.

A test allowing to determine the change point of the variance (in the case when observations belong to the normal law) is proposed in [15] of this test are presented as follows. Let for $k = 1, 2, ..., n-1$

$$w_k = \sum_{i=1}^{k} (x_i - m_x)^2, W_k = \frac{w_n - w_k}{w_k} \frac{k}{n-k},$$

where k corresponds to the required variance change point. If x_i belongs to normal law, then values of $W_k, k = 1, 2, ..., n-1$, belong to corresponding $F_{n-k,k}(W)$ Fisher distributions with $n-k$ and k degrees of freedom.

Next, based on the corresponding distribution functions, we find $\gamma_k = F_{n-k,k}(W_k)$, where γ_k must obey to uniform law under the absence of variance shift.

G-test statistics are given by

$$G = \frac{1}{n-1} \sum_{k=1}^{n-1} \gamma_k, 0 \leq G \leq 1. \tag{6}$$

The hypothesis about absence of variance changes is rejected with significance level α, if $G < G_{\alpha/2}$ or $G > G_{1-\alpha/2}$. In this case value k corresponding to the maximum value $|\gamma_k - 1/2|$, evaluates the desired change point of the variance value in observed series. For $x_1 = m_x$ value $w_1 = 0$, thus $W_1 = \infty$ and $\gamma_1 = 1$.

The type of limit distribution of the statistics (6) is not given in the original material, only percentage points are given. Basing on the results of the statistical simulations we have shown that a good model of the limit distribution of the statistics (6) is a beta distribution of the 1st kind with the density of

$$f(x) = \frac{1}{\theta_2 B(\theta_0,\theta_1)} (\frac{x-\theta_3}{\theta_2})^{\theta_0 - 1} (1 - \frac{x-\theta_3}{\theta_2})^{\theta_1 - 1}$$

and parameter values $\theta_0 = 2.7663, \theta_1 = 2.7663, \theta_2 = 1, \theta_3 = 0$.

Based on this law we can find percentage points $G_{\alpha/2}$ and $G_{1-\alpha/2}$ or p-values.

G-test is also a parametric test. Thus its statistics distributions depend strongly on the type of the law under observation.

2.4 Klotz and Savage rank tests for an absence of variance shifts

Rank tests for detecting the change of the scale parameter (dispersion characteristic) in the unknown point are based on the usage of a family of rank statistics in form [20]

$$S_R = \sum_{i=1}^{n} i a_n(R_i), \tag{7}$$

where R_i are ranks of sampled values in an ordered series of measurements.

Tests differ by the used scores a_n . Their type determines the name of the test. The following scores are commonly used:

- Klotz scores $a_{1n}(i) = U^2_{i/(n+1)}$, where U_γ – is a γ-quantile of standard normal law;
- Savage scores $a_{2n}(i) = \sum_{j=1}^{i} \frac{1}{n-j+1}$.

If the tested hypothesis H_0 is true, then tests with statistics $S_{R,j} = \sum_{i=1}^{n} i a_{jn}(R_i)$, $j = 1, 2$ are free from the distribution and are symmetric with respect to $E[S_{R,j}] = \frac{n+1}{2} \sum_{i=1}^{n} a_{jn}(i)$.

Usually normalized tests with the following statistics are used

$$S^*_{R,j} = \frac{S_{R,j} - E[S_{R,j}]}{\sqrt{D}[S_{R,j}]}, \tag{8}$$

where

$$E[S_{R,1}] = \frac{n+1}{2} \sum_{i=1}^{n} U^2_{i/(n+1)}, E[S_{R,2}] = \frac{n(n+1)}{2};$$
$$D[S_{R,1}] = \frac{n(n+1)}{12} \sum_{i=1}^{n} U^4_{i/(n+1)} - \frac{1}{3n+3}[E[S_{R,1}]]^2;$$
$$D[S_{R,2}] = \frac{n(n+1)}{12}(n - \sum_{j=1}^{n} \frac{1}{j}.$$

Statistics (8) are approximately obeying the standard law. The convergence of the statistics distributions to the standard law was studied in [16, 20].

Statistical simulation research of the distribution of statistics with Klotz scores has shown that for $n > 20$ distribution is well-approximated by the standard normal law. Distribution of the test statistics with Savage scores also matches well with the standard normal law, but only for $n > 30$.

3 Analysis of the test powers

During analysis of test powers for the tests against variance change in an unknown point hypotheses close to the H_0 (in case of normal distribution of random variables) were treated as competitive, when at some point the standard deviation was increased by $5, 10, 15\%$:

$$H_1 : \sigma_1^2 = ...\sigma_k^2 = 1; \sigma_1^{k+1} = ...\sigma_n^2 = 1.1025,$$
$$H_2 : \sigma_1^2 = ...\sigma_k^2 = 1; \sigma_1^{k+1} = ...\sigma_n^2 = 1.21,$$
$$H_3 : \sigma_1^2 = ...\sigma_k^2 = 1; \sigma_1^{k+1} = ...\sigma_n^2 = 1.3225,$$

where $k = n/2$. One competitive hypothesis was considered as more distant:

$$H_4 : \sigma_1^2 = ...\sigma_k^2 = 1; \sigma_1^{k+1} = ...\sigma_n^2 = 4.$$

The presence of a linear trend in the dispersion characteristics of the observed series of random variables (change in scale parameter) in the interval $t \in [0, 1]$ can be simulated according to

$$x_i = \xi_i(1 + ct_i),$$

where $c \in (-1, \infty), t_i = (i - 1) \triangle t, \triangle = 1/n$. True tested hypothesis H_0 corresponds to parameter value $c = 0$.

In case of a periodic trend in the characteristics of dispersion, random values can be simulated, for example, in accordance with the following formula:

$$x_i = \xi_i(1 + d \sin(2k\pi t_i)$$

for $|d| < 1$. In case of a combined trend it can be simulated according to

$$x_i = \xi_i(1 + ct_i + d\sin(2k\pi t_i)$$

for $|d| < 1$, if $c \geq 0$, and for $|d| < 1 + c$, if $c \in (-1, 0)$. The absence of a periodic component of the trend corresponds to the parameter value $d = 0$, and the absence of a linear component corresponds to $c = 0$.

During the analysis of power with respect to linear, periodic, and combined trend in the dispersion characteristics (in variance) of a random variable in the interval $t \in [0, 1]$ the following competitive hypotheses were considered:

$$H_5 : x_i = \xi_i(1 + ct_i), c = 1;\ H_6 : x_i = \xi_i(1 + d\sin(2k\pi t_i), d = 0.8, k = 2;$$
$$H_7 : x_i = \xi_i(1 + ct_i + d\sin(2k\pi t_i), c = 1, d = 0.8, k = 2.$$

At that, $t_i = (i - 1) \triangle t, \triangle t = 1/n$, and random variables x_i have been simulated according to the normal law with parameters m and s.

In the course of work statistical simulation methods (for probabilities of errors of the first kind $\alpha = 0.15, 0.1, 0.05, 0.01$) provided estimations of the capacity of the investigated criteria with respect to the competitive hypotheses H_1, H_2, H_3 and H_4 (corresponding to the shift of the dispersion value), and with respect to the competitive hypotheses H_5, H_6, H_7,corresponding to the presence of a linear or nonlinear trend in the characteristics of the dispersion process.

In the columns of Table 2 tests are ordered by decreasing power $1 - \beta$ according to the power estimations with respect to studied competitive hypotheses with the significance level $\alpha = 0.1$ and sample volume $n = 100$.

For similar competitive hypotheses criteria Hsu tests with H and G statistics as well as Klotz test showed the highest power with respect to the analyzed sets of competitive hypotheses. They showed the ability to detect trend in the dispersion characteristics when it has a 10% increase. Hsu tests with $H-$ and $G-$statistics and Klotz test are also detecting the presence of a linear or periodic trend in the dispersion characteristics (H_0 is distinguished from the hypotheses H_5, H_6).At the same time Cox-Stuart, Savage and Foster-Stuart tests can not detect the presence of a periodic trend in the variance reliably (due to relatively low power against similar enough hypothesis H_6). Unfortunately, none of these tests has shown the ability to detect a mixed trend in the dispersion corresponding to the studied hypothesis H_7. The power with respect to such close hypothesis has been extremely low.

Considered criteria can be placed in order of preference in the following way [22]:

Trend in mathematical expectation

K-inversion, Reversed inversion $\succ$ Inversions$\succ$ Cox-Stewart$\succ$ Autocorrelation test modification$\succ$ Ramachandran-Ranganathan $\succ$ Wald-Wolfowitz, autocorrelation, Dufour-Roy, Moran, Ljung-Box$\succ$ Wald-Wolfowitz rank, Rank Dufour-Roy, Hollin$\succ$ Bartels $\succ$ CUSUM $\succ$ Series Wald-Wolfowitz test $\succ$ Foster-Stewart $\succ$ Number of sign series of the first-order differences.

Trend in variance

$$HsuH - test \succ Klotztest \succ HsuG - test \succ Cox - Stewart \succ$$
$$Foster - Stewarttest \succ Savagetest.$$

Conclusions

Thus, methods of statistical simulation have been used to study the statistics distribution of various parametric and nonparametric tests for randomness and the absence of a trend in the dispersion characteristics; within the framework of developing ISW software an interactive study mode of the distributions of the statistics has been implemented for the case of violation of standard assumptions. A comparative analysis of test powers against some competitive hypotheses has been carried out, and results of such analysis can be used to estimate the desirability of application of particular test. Disadvantages of individual criteria have been noted.

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Table 1: Main advantages and disadvantages of used tests for an absence of trend in mean

№	Test	Advantages	Disadvantages
1	Inversion	High power in respect to linear trend. For $n \geq 30$ discreteness of normalized statistics can be neglected.	The discreteness of normalized statistics must be considered for $n < 30$.
2	Reversed inversion		
3	K-inversion		
4	Cox-Stuart	Power is above the average. For $n \geq 40$ discreteness of normalized statistics can be neglected.	For $n < 40$ discreteness of normalized statistics must be considered.
5	Autocorrelation test modification	Relatively good power.	The difference of normalized statistics distribution from the standard normal law can be neglected only for $n \geq 200$
6	Ramachandran-Ranganathan	Relatively good power.	Statistics distribution have strong dependence on n. Usage of a table of critical values is necessary.
7	Dufour-Roy	The difference of normalized statistics discrete distribution from the standard normal law can be neglected for $n > 17$.	Low power.
8	Autocorrelation	The difference of normalized statistics distribution from the standard normal law can be neglected for $n > 30$.	Low power.
9	Moran		Low power. The difference of statistics distribution from the standard normal law can be neglected only for $n > 50$.
10	Ljung-Box		Low power. Statistics distribution converge very slowly to standard normal law.
11	Wald-Wolfowitz	The difference of normalized statistics distribution from the standard normal law can be neglected for sample sizes $n > 20$.	Low power.

12	Hollin	Average power.	Distribution of the statistics depends on n. The test is nonparametric, yet distribution of the statistics reacts to asymmetry of the observed law.
13	Rank Wald-Wolfowitz	Standard normal law can be used for $n > 10$ as distribution of the proposed modification of normalized statistics.	The power is slightly smaller than one of Dufour-Roy and Wald-Wolfowitz tests. Is equal to rank Dufour-Roy test.
14	Rank Dufour-Roy	For $n > 17$ distribution of the statistics is well-approximated by standard normal law. Discreteness of statistics distribution can be neglected for $n > 10$.	The power is slightly smaller than one of Dufour-Roy and Wald-Wolfowitz tests. Is equal to rank Wald-Wolfowitz test.
15	Bartels	The difference of normalized statistics discrete distribution from the standard normal law can be neglected for $n > 10$.	Low power.
16	Foster-Stuart		High discreteness of statistics distribution, persisting for hign values of n. Usage of assymptotic Student t_n-distribution for evaluation of p-value leads to serious errors. Power against linear trend is below the average. Power against nonlinear trend is low.
17	CUSUM	Good power against linear trend.	Statistics distribution is discrete and it is dependent on n. Very low power against nonlinear trend.
18	Series Wald-Wolfowitz		Normalized statistics distribution is discrete for a long time. Low power.
19	Number of sign series of the first-order differences		Normalized statistics distribution is discrete even for large sample sizes.Extremely low power.

Table 2: Comparative analysis of powers of all tests for randomness and tests for an absence of a trend in variances ($n = 100, \alpha = 0.1$)

№	Against H_1	$1-\beta$	Against H_2	$1-\beta$	Against H_3	$1-\beta$
1	Hsu H	0.156	Hsu H	0.304	Hsu H	0.500
2	Klotz	0.151	Klotz	0.287	Klotz	0.469
3	Hsu G	0.147	Hsu G	0.269	Hsu G	0.430
4	Cox-Stuart	0.123	Cox-Stuart	0.188	Cox-Stuart	0.284
5	Savage	0.110	Foster-Stuart	0.130	Foster-Stuart	0.165
6	Foster-Stuart	0.106	Savage	0.129	Savage	0.159

№	Against H_4	$1-\beta$	Against H_5	$1-\beta$	Against H_6	$1-\beta$
1	Hsu H	1	Hsu H	0.836	Hsu H	0.711
2	Klotz	1	Hsu G	0.818	Klotz	0.678
3	Cox-Stuart	0.997	Klotz	0.807	Hsu G	0.545
4	Hsu G	0.993	Cox-Stuart	0.489	Savage	0.196
5	Foster-Stuart	0.625	Foster-Stuart	0.346	Cox-Stuart	0.143
6	Savage	0.610	Savage	0.246	Foster-Stuart	0.048

№	Against H_7	$1-\beta$
1	Hsu H	0.162
2	Klotz	0.104
3	Savage	0.095
4	Foster-Stuart	0.082
5	Hsu G	0.057
6	Cox-Stuart	0.052