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NONPARAMETRIC GOODNESS-OF-FIT TESTS WHEN CHECKING COMPOSITE HYPOTHESES

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Abstract. Using methods of computer simulation it has been shown that when checking composite hypotheses the distributions of statistics of nonparametric goodness-of-fit tests depend not only on the law $F(x,\theta)$, corresponding to hypothesis H_0, but also on the method of estimating the parameter θ. It has been shown that tests have a maximum power when using maximum likelihood method.

With usage of goodness-of-fit tests the simple hypotheses of the form $H_0: F(x) = F(x,\theta)$, where $F(x,\theta)$ - the cumulative distribution function, with which is checked goodness-of-fit of observed sample, θ - known value of the parameter (scalar or vectorial), and composite hypotheses $H_0: F(x) = F(x,\hat{\theta})$, where $\hat{\theta}$ - the estimate of the parameter calculated from the same sample (if an estimate $\hat{\theta}$ is calculated from other sample, a hypothesis is simple) can be checked. During check from the sample the value S^* of used criterion statistics is calculated. Further, to make conclusion about that, to accept or to reject a hypothesis H_0, it is necessary to know a conditional distribution $G(S|H_0)$ of statistics S when H_0 is valid. And if probability

$$P\{S > S^*\} = \int_{S^*}^{+\infty} g(s|H_0)ds$$

is large enough, at least $P\{S > S^*\} > \alpha$, where $g(s|H_0)$ - conditional density, and α - given significance level (probability of an error of first kind - to reject a true hypothesis H_0), there is no reason to reject a hypothesis H_0. If during the analysis of sample the alternative H_1 is considered: $F(x) = F_1(x,\theta)$, it is connected with conditional distribution $G(S|H_1)$ and probability of an error of second kind β (to accept a hypothesis H_0, while the hypothesis H_1 is correct). The representation of α for a used goodness-of-fit test also uniquely determines β:

$$\alpha = \int_{S_\alpha}^{+\infty} g(s|H_0)ds, \quad \beta = \int_{0}^{S\alpha} g(s|H_1)ds.$$

In this case the more is the power of criterion $1-\beta$, the better it distinguishes appropriate hypotheses.

Nonparametric goodness-of-fit tests (Kolmogorov's–Smirnov's test, Cramer-von-Mises tests) have property of " freedom from distribution ". But when distribution's parameters are estimated those tests lose that property. First, dependence of statistics distributions on the law's form was noted in [1]. Since then many publications were devoted to this problem. However solution was received only in particular situations.

In papers [2, 3] with a help of computer analysis of statistics' distributions $G(S|H_0)$, where S is corresponding statistics, $H_0: F(x) = F(x,\hat{\theta})$ is basic hypothesis, $\hat{\theta}$ is parameter estimate, were constructed simple and sufficiently exact approximations for $G(S|H_0)$ with the different laws of random variable $F(x,\theta)$ and maximum likelihood method of parameters estimation.

Further studies have shown that distributions of nonparametric tests' statistics when checking composite hypotheses essentially depend not only on the laws form $F(x,\theta)$, on the estimated parameter's find, on the number of estimated parameters, on the specific parameter's values, but also on the method of estimating.

In the present moment the problems of application of nonparametric goodness-of-fit tests when checking composite hypotheses in connection with distinction in methods of estimation are not reflected in the scientific publications. Russian and international standards of statistical method’s applications don't contain recommendations to application of nonparametric tests when checking composite hypotheses. Therefore application of those tests in practice often leads to incorrect conclusions.

The distributions of statistics of nonparametric goodness-of-fit tests when checking composite hypotheses depend on character of this composite hypothesis. The law of distribution of a statistic $G(S|H_0)$ is influenced by a lot of factors, determining "complexity" of a hypothesis: the form of the observed law of distribution $F(x,\theta)$, appropriate to a true hypothesis H_0; the type of the estimated parameter and the amount of estimated parameters of the law $F(x,\theta)$, and in some situations a specific value of the parameter, as in case of gamma - distribution; a used method of parameter’s estimation. For small sample sizes n the distribution $G(S_n|H_0)$ depends also on from n. Though truth essential depedence from n is observed only for small sample sizes.

Using methods of statistical simulation, we researched how the size of observed sample tells on distributions of statistics of nonparametric goodness-of-fit tests for simple and composite hypotheses. As things turned out already for $n \geq 15 \div 20$ the distribution $G(S_n|H_0)$ is close enough to limiting $G(S|H_0)$, and the dependence from n can be neglected.

In case of giving a specific alternative (competing hypothesis H_1, to which distribution $F_1(x,\theta)$ corresponds) cumulative distribution function of a statistic $G(S|H_1)$ also depends on all enumerated factors. Moreover as distinct from $G(S|H_0)$, the distribution of a statistic $G(S|H_1)$ with a true hypothesis H_1 very strongly depends on the sample size n: due to this fact the ability of criteria to distinguish close hypotheses increase with growth of n the power of criteria increase.

In present work applying the methods of computer simulation we researched distributions $G(S|H_0)$ and $G(S|H_1)$ of statistics of Kolmogorov’s test and ω^2 of Mises in case of usage of various methods of estimation for various laws $F(x,\theta)$, most frequently used in applications.

In criteria such as Kolmogorov’s test (Kolmogorov-Smirnov’s test) most frequently is used the following statistic [4]

$$S_K = \frac{6nD_n + 1}{6\sqrt{n}},$$

where $D_n = \max(D_n^+, D_n^-)$, $D_n^+ = \max\limits_{1\leq i\leq n}\left\{\frac{i}{n} - F(x_i,\theta)\right\}$, $D_n^- = \max\limits_{1\leq i\leq n}\left\{F(x_i,\theta) - \frac{i-1}{n}\right\}$, n - sample size, $x_1, x_2, \ldots, x_n$ - sample values, ordered by increase, $F(x)$ - function of the law of distribution, goodness-of-fit with which is checked. The distribution of variable S_K for simple hypothesis in a limit obeys the Kolmogorov's law $K(S)$ [4].

Statistic ω^2 of Mises's test (Cramer-von-Mises-Smirnov test)

$$S_\omega = n\omega_n^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left\{F(x_i,\theta) - \frac{2i-1}{2n}\right\}^2$$

with a simple hypothesis obeys the distribution $a1(S)$ [4].

Distributions of statistics in this case were compared when using of maximum likelihood estimates (MLE) and MD-estimations. When evaluation of MD-estimations appropriate distance between empirical and theoretical distributions is minimized. When using a statistic S_K as an estimate of the parameter vector θ the values minimizing this statistic are selected: $\hat{\theta} = \arg\min_{\theta} S_K$ (MD-estimation S_K). Similarly, when using a statistic S_ω it (MD-estimate S_ω) is minimized on θ: $\hat{\theta} = \arg\min_{\theta} S_\omega$.

In fig. 1 the distributions $G(S_n|H_0)$ of Kolmogorov's statistics S_K are represented when checking a composite hypothesis with usage of maximum likelihood method for estimation of two parameters of the law, corresponding to a hypothesis H_0 (1 - normal with density $f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$, 2 - logistics $f(x) = \frac{\pi}{\sigma\sqrt{3}}\exp\left\{-\frac{\pi(x-\mu)}{\sigma\sqrt{3}}\right\} \Big/ \left[1+\exp\left\{-\frac{\pi(x-\mu)}{\sigma\sqrt{3}}\right\}\right]^2$, 3 - Laplace $f(x) = \frac{\theta_0}{2}e^{-\theta_0|x-\theta_1|}$, 4 - least value $f(x) = \frac{1}{\theta_0}\exp\left\{\frac{x-\theta_1}{\theta_0} - \exp\left(\frac{x-\theta_1}{\theta_0}\right)\right\}$, 5 - Cauchy with density $f(x) = \frac{\theta_0}{\pi[\theta_0^2 + (x-\theta_1)^2]}$. In fig. 2 the distributions $G(S_n|H_0)$ of the same statistic S_K are represented when checking the same hypotheses, but with usage of MD-estimates S_K.

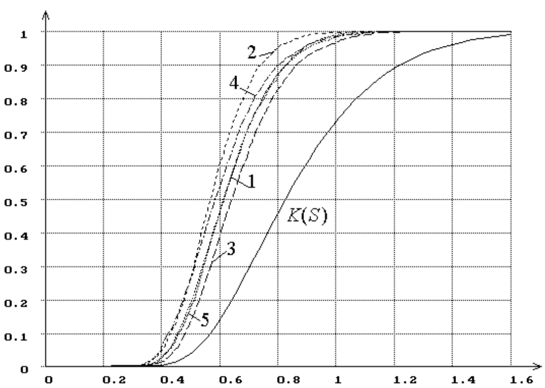

Fig. 1.

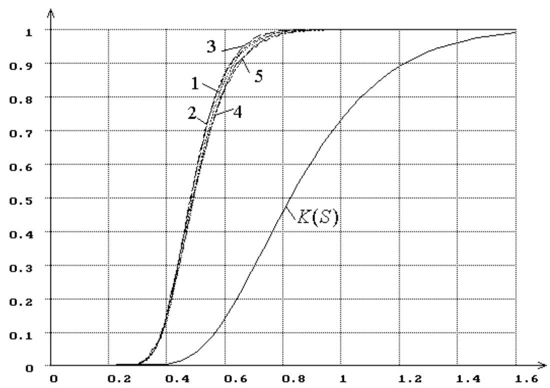

Fig. 2.

In fig. 3 the distributions of a statistic S_ω for similar hypotheses H_0 are presented while using MLE, and in fig. 4 - while using MD-estimates minimizing a statistic S_ω on parameters.

When using of MD-estimates minimizing a statistic of criterion, the empirical distributions $G(S_n|H_0)$, corresponding to various hypotheses H_0, have a minimum scatter, that allows to speak about a certain “freedom from distribution” of criteria. If we base only on this fact, it would seem, that only such methods of estimation should be applied when checking composite hypotheses.

However research of a power of considered criteria for various methods of estimation has shown, that the nonparametric criteria with close alternatives have maximum power in case of a parameter estimation by maximum likelihood method. In fig. 5 distribution $G(S_n|H_0)$ and distribution $G(S_n|H_1)$ of the statistics S_ω of Cramer-von-Mises-Smirnov's test are represented depending on n for composite hypothesis (H_0 - normal distribution, H_1 - logistic) with n = 20, 100, 500, 1000, when using MLE, and in fig. 6 - when using of MD-estimates S_ω.

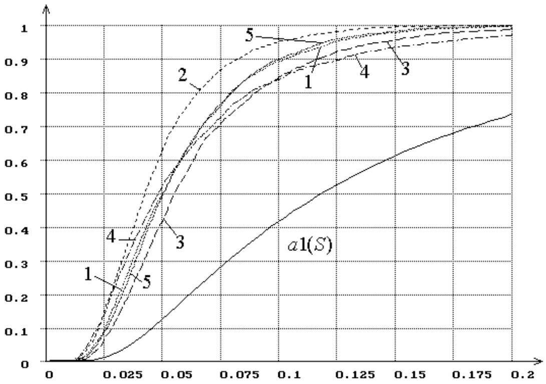

Fig. 3.

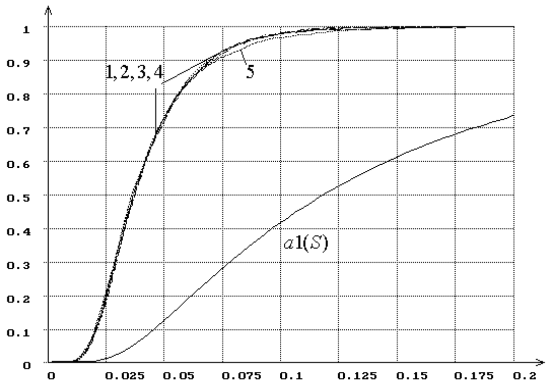

Fig. 4.

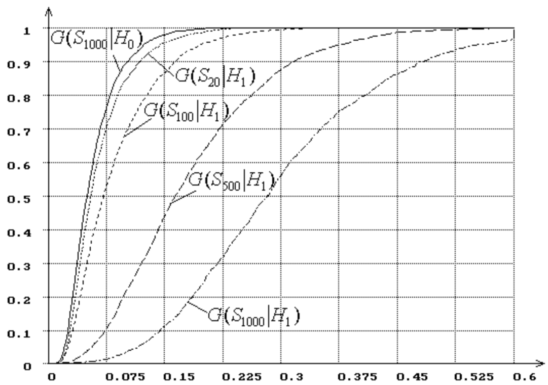

Fig. 5.

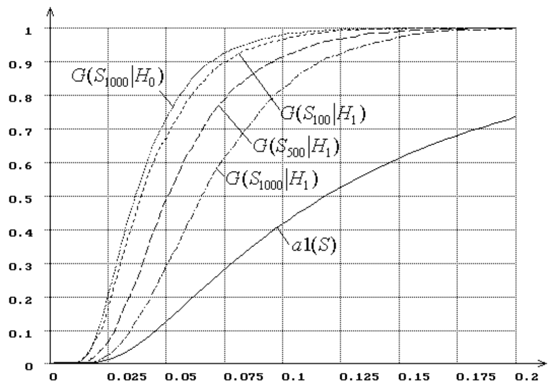

Fig. 6.

In the work the distributions $G(S|H_0)$ and $G(S|H_1)$ for various hypotheses H_0 and various close H_1 were researched depending on sample size. On the grounds of the obtained results development of "Rules for checking goodness-of-fit of experienced distribution with theoretical one" when checking composite hypotheses with nonparametric tests becomes possible.

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